Generational War on Inflation?

Optimal Inflation Rates for the Young and the Old*

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Abstract

How does population aging affect the political decision-making regarding the inflation target? Using an overlapping generations New Keynesian model, we analyze how demographic factors affect the inflation target rates preferred by young and old households, respectively. Given a demographic structure, the earnings redistribution channel leads young and old households to have different preferences over the inflation target: the optimal target rate for the average old household is negative, whereas that for the average young household is close to zero. Hence, the composition effect of population aging on the population-weighted average of these optimal rates is negative. However, the overall effect is either positive or only marginally negative, depending on whether the transition to a new steady state is taken into account, and there is no strong negative association between the population-weighted average of the optimal inflation rates and population aging. The analysis reveals that greater longevity and lower birth rates make the preferences of young and old households more similar, either directly through utility functions or indirectly through a lower real interest rate.

Keywords: Optimal inflation rates; Societal aging; Heterogeneous agents

JEL codes: E31; E52; J11

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1 Introduction

Population aging is one of the biggest problems many economies are facing. An increasing age-dependency ratio poses numerous challenges to society, such as the sustainability of pay-as-you-go pension systems, that of government debt, and private expenditures for health care due to chronic diseases. A prime example of aging economies is Japan: its working-age population peaked in 1995 and has been declining steadily since; the population share of people aged 65 and over reached 29.1 percent in 2021 and is expected to rise further. An interesting observation about Japan is that inflation began to decline around the mid-1990s, which is also when the working-age population started decreasing, and has remained low and stable for more than two decades since. Is this a mere coincidence, or is there an underlying mechanism that connects the two phenomena?

In this paper, we examine whether population aging leads society to prefer lower inflation or even deflation. If old households prefer lower inflation than do young households, then utilitarian welfare might be maximized by setting a low inflation target in an aging economy, through the composition effect. The welfare-maximizing target rate may even be negative if old households prefer deflation. However, population aging may bring about other changes to the economy that affect the inflation target rates preferred by young and old households. If population aging makes households prefer higher inflation, then the welfare-maximizing inflation target rate may rise in response to rapid population aging, overturning the composition effect.

To analyze how agents in different demographic groups prefer different inflation target rates and how their preferred target rates change in an aging economy, we use a tractable overlapping-generations New Keynesian (OLG-NK) model in which the inflation target has redistribution effects. The model builds on Fujiwara and Teranishi (2008), which enrich the overlapping generations model in Gertler (1999) with nominal rigidities.[1] Households age and die stochastically, and their labor endowment and effective discount factors decline with their age: when they are young they work and save for retirement, and when they become old they can no longer work and have a higher marginal propensity to consume. Inflation is not neutral due to price stickiness and has the redistribution effect because young and old households have different sources of income: young households receive both labor income and asset income, whereas old households receive only asset income. For example, negative inflation not only distorts the aggregate production but also redistributes resources from workers to asset holders through lower wages and higher corporate profits that are distributed

[1]The overlapping generations model in Gertler (1999) can be considered as the generalized Blanchard-Yaari model *à la* Blanchard (1985) and Yaari (1965).

as dividend to asset holders.[2]

We compute the optimal inflation target in the spirit of Schmitt-Grohe and Uribe (2010) with a *politico-economic* consideration following Bassetto (2008), who studies the inter-generational conflicts in tax policy in overlapping generations. The central bank is assumed to achieve constant inflation under the strict inflation targeting regime. The optimal inflation target rates are computed as the utility-maximizing inflation targets for young and old households, respectively. We are interested in (1) whether and how much young and old households differ in their preferences over the inflation target and (2) whether and how much population aging affect the optimal inflation target rates for young and old households.

Our first result is that, given the demographic structure, young and old households in the model indeed have different preferences over the inflation target, with the old preferring deflation while the young preferring near-zero inflation target. Hence, population aging has a negative composition effect on the population-weighted average of the optimal inflation targets. This result is obtained both in the steady-state analysis and in the analysis with transition. The representative (i.e., average) young household's steady-state utility is maximized when the inflation target is approximately zero, whereas the representative old household's is maximized when the inflation target is slightly negative and approximately -0.2 percent per annum. We also conduct an analysis taking into account a transition from a zero-inflation steady state. Old households still prefer lower inflation than young households, just as they do in the steady-state analysis, but both the representative young and old households in the initial period prefer *lower* inflation targets than they do in the steady-state analysis. This is because those who are present in period 0 can entertain front-loaded consumption when the economy transits to a steady state with a lower rate of inflation, where capital accumulation is distorted downward. Such front-loading of consumption is made possible at the expense of lower consumption of future newborns.

It is worth mentioning that there is a huge heterogeneity in young households' preferences toward the inflation target, due to the heterogeneity in their financial wealth. Because newborns begin their lives with no financial wealth in our model, those who have not worked for long enough have little to no financial wealth. They prefer inflation because human wealth is much more important for them than financial wealth. In contrast, there are also young households who have accumulated a large amount of financial wealth because they have been working for long and have not yet retired. They prefer deflation because they benefit from redistribution from human wealth to finan-

[2] This redistribution effect is related to the earnings heterogeneity channel coined by Auclert (2019). The difference is that we consider the effect of a long-run inflation target whereas Auclert (2019) focuses on a short-run policy shock.

cial wealth. In our model, young households accumulate financial wealth relatively quickly, and the majority of them prefer negative inflation.

Our second result is that, despite the negative composition effect, population aging and the population-weighted optimal inflation target do not exhibit a strong negative association. The two key drivers of population aging — higher life expectancy and a lower birth rate — both act to increase the optimal inflation targets for the representative young and old households in period 0, and their positive effects dominate the negative composition effect. Hence, the population-weighted optimal inflation target and population aging are positively associated. Their relation is quantitatively weak: with reasonable parameters, population-weighted optimal inflation rate ranged only from -0.25 to 0 percent per annum. If we are concerned only with the steady-state welfare, the population-weighted optimal inflation target declines with population aging but only marginally. Overall, the quantitative importance of population aging in rationalizing declining inflation rate in Japan is limited.

We further investigate the mechanism behind the second result. Our tractable model allows us to analytically characterize preference heterogeneity between young and old households and to examine how population aging affects the heterogeneity. The analysis reveals two key determinants for the preference heterogeneity in the steady state: the ratio of marginal propensities to consume of young and old households is determined by the survival probability of old households and the real interest rate. Old households have a higher marginal propensity to consume than young households because of the possibility of death. Greater longevity, i.e., a higher survival probability, reduces the marginal propensity to consume of old households, making it more similar to that of young households. Aggregate savings increase and the real interest rate falls. A lower real interest rate induces both young and old households to save more, leading to a further decline in the real interest rate. We find that both greater longevity and lower birth rates lead to a lower real interest rate and higher aggregate savings. Hence, population aging makes both young and old households own more financial wealth, blurring the distinction between workers and retirees.

In this paper we assume that the central bank achieves constant inflation. This assumption allows us to analyze households' preferences are over a single dimensional object, the inflation target rate, instead of a sequence of inflation rates. A key assumption of our analysis is that the inflation target is not neutral even in the long run. In the New Keynesian literature, trend inflation is found to be nonneutral even in a steady state, as shown for the Calvo (1983) contract in Ascari (2004) and for Rotemberg (1982) adjustment costs in Bilbiie et al. (2014), and as comprehensively analyzed in Schmitt-Grohe and Uribe (2010). The long-run inflation rate affects mark-ups through nominal rigidities and has impacts on real variables.

The remainder of the paper is structured as follows. Section 2 describes the model. In Section 3, we show how the optimal inflation rates are different between the young and the old and how the demographic structure affect them. Section 4 examines how population aging affects the optimal inflation target rates. Section 5 concludes.

1.1 Related literature

Optimal long-run inflation Several existing papers have considered optimal long-run inflation rate. Coibion et al. (2012) analyze optimal long-run inflation rate in the presence of the zero lower bound and find that a positive but low inflation is optimal. Kim and Ruge-Murcia (2011) and Kim and Ruge-Murcia (2019) analyze optimal target inflation rate a model with downward nominal wage rigidity. We also investigate optimal long-run inflation rate as in these papers, in a setting with price stickiness and overlapping generations.

There are recent papers that pay attention to the role of heterogeneity in optimal monetary policy: Adam and Weber (2019) consider heterogeneity in firms' productivity trends over life-cycle; Mineyama (2022) considers a model with downward nominal wage rigidity and heterogeneity in labor productivity; and Menna and Tirelli (2017) focus on asset composition heterogeneity. Our paper is in line with this literature by focusing on demographic heterogeneity.

In these papers, relative price dispersion or the price adjustment costs constitute welfare costs of inflation. Relative price dispersion is one of the costs of inflation in Coibion et al. (2012) and Mineyama (2022) in considering optimal inflation rate, although Nakamura et al. (2018) do not find a strong link between inflation rate and relative price dispersion,[3] and the cost of inflation is price adjustment cost in Kim and Ruge-Murcia (2011) and Kim and Ruge-Murcia (2019). In our model, the cost of long-run inflation is also the relative price dispersion or the price adjustment costs, depending on the price setting assumption.

Demographics and the natural rate of interest An aging population and low inflation rates are not phenomena intrinsic only to Japan and are now observed in many developed economies, leading several researchers to investigate the possible causal relationship between inflation dynamics and demographic changes. Carvalho and Ferrero (2014) and Fujita and Fujiwara (2016) discuss how societal aging can lead to a decline in the natural rate of interest, which exerts downward pressure on inflation with

[3] Phaneuf and Victor (2019) provide a New Keynesian model that explains the week link between inflation rate and price dispersion.

insufficient monetary policy responses. Carvalho and Ferrero (2014) focus on the demand channel, or *consumption-saving heterogeneity*. Longer life expectancy (longevity) induces higher saving rates for self-insurance. Such a saving-for-retirement motive can account for roughly 30 percent to 50 percent of the decline in real interest rates in Japan. The decline in fertility rate, however, does not have large impacts. In contrast, Fujita and Fujiwara (2016) quantify the impact of the supply channel, or *skill (productivity) heterogeneity*. The changes in the demographic structure induce significant low-frequency movements in *per-capita* consumption growth and the real interest rate through changes in the composition of skilled (old) and unskilled (young) workers. This mechanism can account for roughly 40 percent of the decline in the real interest rate observed between the 1980s and 2000s in Japan. The key is the declining fertility (labor participation) rate.

Short-run shocks and redistribution Our analysis is normative and focuses on heterogeneous households' preferences over the long-run inflation target, but there are studies that instead focus on a positive analysis of redistribution by a short-run monetary policy shock. Doepke and Schneider (2006) explore the redistribution effect of inflation. Since the old households tend to own more nominal financial assets, they are more vulnerable to unanticipated inflation. On the other hand, surprise inflation can be beneficial to the young households because they tend to be borrowers and their debt are often nominal debt. Hence, a short-run change in inflation differently affects households of different ages, and societal preference over inflation is expected to depend on the demographic structure. Auclert (2019) examines the redistribution effects of monetary policy on aggregate consumption in the economy populated with households of different marginal propensities to consume. He identifies three channels through which an unanticipated short-run monetary policy shock causes redistribution: an *earning heterogeneity* channel, a *Fisher channel*, and an *interest rate exposure* channel. Auclert (2019) finds that all three channels amplify the effects of monetary policy. There are many other studies pointing out the heterogeneous impacts of monetary policy. Examples include Fujiwara and Teranishi (2008), Kaplan et al. (2018), Debortoli and Galí (2017), Wong (2016) and Eichenbaum et al. (2022). Our analysis differs from theirs in that we focus on the effects of a long-run inflation target, not those of a short-run surprise shock, and that we focus mainly on the earnings heterogeneity channel. [4]

[4] From a normative perspective, Sheedy (2014) shows that nominal GDP targeting is desirable in an economy with nominal financial contracts, since it can improve risk sharing.

Bullard et al. (2012) construct an overlapping generations model with two assets, capital and money, but without nominal rigidities. If old agents have more influence on political decision making, relatively low inflation is chosen because lower inflation reduces the opportunity cost of holding money and money becomes relatively more attractive, thus reducing capital accumulation. This raises interest rates, which is preferred by the old since they rely more on capital income than labor income.

Katagiri et al. (2019) also examine the negative correlation between inflation and aging from a politico-economic perspective, using a two-period OLG model with neither a short- nor a long-run Phillips curve. The key mechanism in their paper is the Fiscal Theory of the Price Level (FTPL): the government issues nominal debt and population aging affects inflation through its effects on the tax base. A sequence of myopic government maximizes weighted average of the current households' utility., and they focus on a Markov perfect equilibrium. They also find that the effect of population aging on equilibrium inflation depends on whether aging is due to an increase in longevity or a decline in the birth rate. In contrast, the government does not issue nominal debt and our analysis is concerned with policy-making under *monetary dominance*.

Gornemann et al. (2016) characterize heterogeneous preferences among households about the monetary policy reaction function. They use a heterogeneous-agents New Keynesian model with a richer set of heterogeneity and idiosyncratic shocks: in addition to stochastic retirement and death that are present in our model, households in their model are heterogeneous in their education level and preference discount factor and are also subject to unemployment/employment and earning loss shocks. A key difference between this paper and theirs is that their focus is on the short-run whereas ours is on the long-run: they focused on the Taylor rule coefficients on inflation and the unemployment rate taking the central bank's target inflation as given, whereas we focus on the target rate of inflation under the strict inflation targeting.

Braun and Ikeda (2025) investigate how monetary policy shocks affect households of different ages, using an overlapping generations New Keynesian model that is calibrated to the Japanese data. They find that the responses of consumption and various kinds of incomes to a monetary policy shock are heterogeneous across ages. Their focus is on the short run and the analysis is descriptive, i.e., the households' responses to a temporary shock to the nominal interest rate, whereas our focus is on the long run and our analysis is normative.

2 The model

In order to investigate the effects of societal aging on the optimal inflation rate, we employ the overlapping generations (OLG) model used in Fujiwara and Teranishi (2008), which extends the analytical framework in Gertler (1999) to incorporate nominal rigidities and monetary policy. The biggest advantage of this framework is its tractability. Unlike the standard overlapping generations model, individuals age and die stochastically. There are two age groups, the young and the old. A consumer is born in the young group, the transition from the young to the old occurs randomly, and each consumer in the old group dies stochastically. Thanks to the consumer preference specification, the model permits aggregation within each age group, and the age-wealth distribution matters for the aggregate dynamics only through two variables: wealth held by the young and that held by the old. This property facilitates computation and enables us to understand the mechanisms behind non-zero optimal inflation rates for heterogeneous agents more intuitively. There is no aggregate uncertainty and we assume perfect foresight for aggregate fluctuations throughout this paper.

There are six agents in this model economy: two types of consumers — the young and the old; final good producers; intermediate goods producers; a capital producer (financial intermediary); and the central bank. In what follows we explain their behavior one by one.[5]

2.1 Consumers

There are two types of consumers, namely, the young and the old. They differ in their labor productivity, exogenous flow incomes (profits, taxes, and transfers), and their effective preference discount factors. Non-zero inflation not only distorts the equilibrium allocation but also redistributes income between the young and the old through changes in mark-up. Income redistribution across different age groups has macroeconomic consequences because these groups have different marginal propensity to consume out of wealth due to the difference in their effective discount factors.

In the benchmark model, young agents inelastically supply one unit of labor, whereas old agents never work and, therefore, receive no labor compensation. One interpretation is that both young and old agents are endowed with one unit of labor that is supplied inelastically but the labor productivity is one for the young and zero for the latter. Hence, aging in this model is a synonym for retirement and for a negative labor productivity shock at an individual level.

[5] For details of the derivation, see also Gertler (1999) and Fujiwara and Teranishi (2008).

2.1.1 Population dynamics

Each young agent faces a constant probability ω to remain young and $1-\omega$ to become old, while each old agent remains in the population with the survival probability γ and dies with probability $1-\gamma$. Let N_t^y and N_t^o denote the population of young agents and that of old agents in period t, respectively. The number of newborns in period $t+1$ is proportional to the young population in t, N_t^y, with the proportionality (the birth rate) being denoted by b. Thus, the population dynamics are given by

$$N_{t+1}^y = (b+\omega)N_t^y,$$

and

$$N_{t+1}^o = \gamma N_t^o + (1-\omega)\,N_t^y.$$

The growth rate of young population, n, is given by

$$n := b+\omega-1.$$

Given these laws of motion, the ratio of the number of old to that of young agents, denoted by Γ_t, evolves as

$$\Gamma_{t+1} := \frac{N_{t+1}^o}{N_{t+1}^y} = \frac{\gamma N_t^o + (1-\omega)\,N_t^y}{bN_t^y + \omega N_t^y} = \frac{\gamma}{b+\omega}\Gamma_t + \frac{1-\omega}{b+\omega}.$$

We focus on the stationary age distribution, in which the ratio of the number of old over that of young agents remain constant:

$$\Gamma = \frac{1-\omega}{b+\omega-\gamma}.$$

Under the stationarity assumption, the entire population also grows at the rate n.

2.1.2 Old

Consider an agent who is old in period t, entered in period t with Z_{t-1}^o units of nominal asset (including the interest), and is maximizing her utility from period t on. The utility is recursively determined by a deterministic consumption sequence $\{C_s^o\}_{s\geq t}$ from period t on:

$$V_s^o = \left\{(C_s^o)^\rho + \beta\gamma\left(V_{s+1}^o\right)^\rho\right\}^{\frac{1}{\rho}}, \quad \text{for all } s \geq t.$$

Here C_s^o is a consumption level in period s conditional on being alive, and V_s^o is the continuation lifetime utility from period s on that is also conditional on survival. An old agent in period t therefore maximizes V_t^o. The discount factor is denoted by β, and ρ is the inverse of the intertemporal elasticity of substitution that satisfies $\rho \leq 1$ and $\rho \neq 0$. The survival probability, γ, effectively modifies the discount factor, making the old agents less patient than the young who discount their future utility only by β.

An old agent's flow income consists only of the sum of the transfer (or tax) from the government, because her labor income is zero. The flow income of the old is assumed to be independent of an individual history such as when the agent was born, when she switched from young to old, and her decisions that were made in the past, and thus is denoted simply by D_t^o.

Old agents save through mutual funds. The gross nominal return on the mutual funds between period s and $s+1$ is denoted by R_s. Old agents have access to perfect annuity market: in exchange for giving funds to the mutual fund issuer if they die, old agents receive the nominal return R_s/γ per unit if they survive next period. It is optimal for an old agent to make such an arrangement for all the funds she possesses so that, conditional on survival, her savings yields the gross nominal rate of return of R_s/γ.

Therefore, the flow budget constraints are given by:

$$\frac{Z_s^o}{R_s/\gamma}\frac{1}{P_s} = \frac{Z_{s-1}^o}{P_s} - C_s^o + D_s^o, \quad \text{for all } s \geq t.$$

Here Z_s^o is the nominal asset position including the interest at the beginning of period $s+1$. Therefore, it is discounted by the rate of return on annuity and also divided by the period-s price level in the period-s budget constraint.

The old in period t also face non-negativity constraints for asset, i.e. $Z_s^o \geq 0$ for all $s \geq t$ in order to prevent them from dying with unpaid debt. However, we assume throughout the paper that the non-negativity constraints never bind. [6]

Characterization Notice that the old's problem above depends on an individual history only through the accumulated asset. This implies that all old agents in period t face the same problem but with different initial nominal asset holdings Z_t^o.

The solution to the old's problem is simple. Let H_t^o be the present discounted value of flow incomes from t on, $\{D_s^o\}_{s\geq t}$, which is defined recursively:

$$H_s^o = D_s^o + \frac{1}{r_s/\gamma} H_{s+1}^o, \quad \text{for all } s \geq 0, \tag{1}$$

[6] This is an important assumption to obtain the linear aggregation result below. The aggregation result fails if the borrowing constraint binds. See Waki (2022).

where $r_s = R_s P_s / P_{s+1}$ denotes the gross real interest rate. As in the literature, we call H_t^o human wealth, though the flow incomes that are taken into account are not earned incomes. We assume that the sequence $\{H_t^o\}_{t=0}^{\infty}$ is finite, which is true in a general equilibrium.

We refer to the sum of financial and human wealth, $Z_{t-1}/P_t + H_t^o$, as wealth. Then, both the optimal consumption and the maximized value are linear in wealth:

$$\text{(Consumption rule)} \quad : \quad C_t^o = mpc_t^o \times \left(\frac{Z_{t-1}^o}{P_t} + H_t^o \right), \tag{2}$$

$$\text{(Value function)} \quad : \quad V_t^o = (mpc_t^o)^{\frac{\rho-1}{\rho}} \times \left(\frac{Z_{t-1}^o}{P_t} + H_t^o \right), \tag{3}$$

where $\{mpc_t^o\}$ is a deterministic sequence that satisfies:

$$\left(\frac{mpc_t^o}{1 - mpc_t^o} \right)^{\rho-1} = \beta \gamma^{1-\rho} r_t^{\rho} \left(mpc_{t+1}^o \right)^{\rho-1}, \tag{4}$$

for all $t \geq 0$ and the terminal condition $\lim_{t\to\infty} mpc_t^o = 1 - \left(\beta\gamma^{1-\rho} r^{\rho}\right)^{1/(1-\rho)}$, where we assume that $r = \lim_{t\to\infty} r_t$ and that $\left(\beta\gamma^{1-\rho} r^{\rho}\right)^{1/(1-\rho)} < 1$. The variable mpc_t^o denotes the fraction of wealth that are consumed, and hence represents the old's marginal propensity to consume out of wealth. In Appendix A we verify that the value function and the decision rules above are indeed optimal.

From the budget constraint, the next period's financial wealth is given by

$$\text{(Financial wealth)} : \frac{Z_t^o}{P_{t+1}} = \frac{r_t}{\gamma} \left\{ \frac{Z_{t-1}^o}{P_t} + D_t^o - C_t^o \right\} \tag{5}$$

where C_t^o is determined by (2).

2.1.3 Young

Now consider an agent who is young at time t, entered in period t with Z_{t-1}^y units of nominal asset including the interest, and is maximizing her utility from period t on. The utility is again recursively determined but now stochastic aging needs to be incorporated:

$$V_t^y = \left\{ (C_t^y)^{\rho} + \beta \left[\omega V_{t+1}^y + (1-\omega) V_{t+1}^o \right]^{\rho} \right\}^{\frac{1}{\rho}}.$$

Here V_t^y is the young agent's lifetime utility from period-t onward. This preference specification is called the RINCE (RIsk Neutrality and Constant Elasticity of Substitution) preferences (Farmer, 1990), which is a special case of the Epstein and Zin (1989) preference with risk neutrality. Due to risk neutrality, the above preference aggregator depends on the age-dependent continuation utilities, V_{t+1}^{y} and V_{t+1}^{o}, only through their expected value, $\omega V_{t+1}^{y} + (1-\omega)\, V_{t+1}^{o}$.

Young agents' flow budget constraint differs from old's in three respects. First, young agents receive labor income, whereas old agents do not. Second, there is no insurance market for stochastic aging for the young, and the young can save only through the risk-free mutual funds with the gross rate of return given by R_s. In contrast, old agents can save through annuity that yields the gross rate of return of R_s/γ. Third, the non-labor flow income that young agents receive from the government may be different from the old's. The flow budget constraints for young agents are given by:

$$\frac{Z_s^y}{R_s}\frac{1}{P_s} = \frac{Z_{s-1}^y}{P_s} - C_s^y + D_s^y + \frac{W_s}{P_s}, \quad \text{for all } s \geq t.$$

Characterization For the young households, the marginal propensity to consume out of wealth, mpc_t^y, is characterized by the following recursion:

$$\left(\frac{mpc_t^y}{1-mpc_t^y}\right)^{\rho-1} = \beta r_t^{\rho}\left(\omega + (1-\omega)\left(\frac{mpc_{t+1}^o}{mpc_{t+1}^y}\right)^{\frac{\rho-1}{\rho}}\right)^{\rho}(mpc_{t+1}^y)^{\rho-1}, \tag{6}$$

for all $t \geq 0$.

Thanks to the RINCE preference, the solution to the young's problem is also linear in appropriately defined wealth. Let H_t^y be the expected present discounted value of flow incomes from t on,$\{D_s^y + W_s/P_s\}_{s\geq t}$, which is defined recursively:

$$H_s^y = D_s^y + \frac{W_s}{P_s} + \frac{\omega(mpc_{s+1}^y)^{\frac{\rho-1}{\rho}} H_{s+1}^y + (1-\omega)(mpc_{s+1}^o)^{\frac{\rho-1}{\rho}} H_{s+1}^o}{\left\{\omega(mpc_{s+1}^y)^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc_{s+1}^o)^{\frac{\rho-1}{\rho}}\right\} r_s}, \quad \text{for all } s \geq 0. \tag{7}$$

We assume that the sequence $\{H_t^y\}_{t=0}^{\infty}$ is finite.

The optimal consumption and the maximized value are again linear in wealth:

$$\text{(Consumption rule)} \;:\; C_t^y = mpc_t^y \times \left(\frac{Z_{t-1}^y}{P_t} + H_t^y\right), \tag{8}$$

$$\text{(Value function)} \;:\; V_t^y = (mpc_t^y)^{\frac{\rho-1}{\rho}} \times \left(\frac{Z_{t-1}^y}{P_t} + H_t^y\right). \tag{9}$$

From the budget constraint, the next period's financial wealth is given by

$$\text{(Financial wealth)} : \frac{Z_t^y}{P_{t+1}} = r_t\left\{\frac{Z_{t-1}^y}{P_t} + D_t^y + \frac{W_t}{P_t} - C_t^y\right\} \tag{10}$$

where C_t^y is determined by (2). Again, optimality of the value function and the decision rules above is shown in Appendix A.

2.1.4 Aggregation

Because the sequence $\{mpc_t^o\}$ is common for all old agents and $\{mpc_t^o\}$ for all young agents, we can linearly aggregate consumption, savings, and the value within each age group.

In particular, the consumption decision rules and the value functions are straightforward to aggregate. For the old agents, we read variables in equations (1), (2), and (3) as aggregate variables for the old population, i.e. C_t^o is the total consumption of the period-t old, Z_{t-1}^o is the total nominal financial wealth of the period-t old at the beginning of period t, D_t^o is the total flow income of the period-t old, H_t^o is the present discounted value of flow incomes of the period-t old, and V_t^o is the utilitarian value of all period-t old agents. For the same reason, we also read variables in equations (7) to (9) as aggregate variables for the young population, i.e. C_t^y is the total consumption of the period-t young, Z_{t-1}^y is the total nominal financial wealth of the period-t young at the beginning of period t, D_t^y is the total flow income of the period-t young, H_t^y is the expected present discounted value of flow incomes of the period-t young, and V_t^y is the utilitarian value of all period-t young agents. When aggregating, the period-s per-person labor income W_s/P_s in equations (7) and (10) needs to be replaced with the total labor income of the young, $W_s N_s^y / P_s$. The dynamics of *aggregate human wealth* for the old and for the young are therefore given by: for all $t \geq 0$,

$$H_t^o = D_t^o + \frac{1}{r_t/\gamma} H_{t+1}^o, \tag{11}$$

$$H_t^y = D_t^y + \frac{W_t}{P_t} N_t^y + \frac{\omega (mpc_{t+1}^y)^{\frac{\rho-1}{\rho}} H_{t+1}^y + (1-\omega)(mpc_{t+1}^o)^{\frac{\rho-1}{\rho}} H_{t+1}^o}{\left\{ \omega (mpc_{t+1}^y)^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc_{t+1}^o)^{\frac{\rho-1}{\rho}} \right\} r_t}. \tag{12}$$

The dynamics of financial wealth, however, cannot be the same as in equation (5) for the old or in equation (10) for the young. On the one hand, the right-hand side of equation (5) is equal to the nominal value of assets held at the beginning of period $t+1$ by the surviving old, i.e. those who were old in period t and also alive in $t+1$, and therefore does not take into account those who were young in period t and became old in $t+1$. On the other hand, the right-hand side of equation (10) is equal to the nominal value of assets held at the beginning of period $t+1$ by those who were young in period t, regardless of their age in $t+1$. Hence, when aggregating, the $1-\omega$ fraction of the right-hand side of equation (10) needs be moved to the old's hands in $t+1$. The dynamics of *aggregate financial wealth* for the old and for the young are therefore given by:

$$\frac{Z_t^o}{P_{t+1}} = r_t\left\{\frac{Z_{t-1}^o}{P_t} + D_t^o - C_t^o\right\} + (1-\omega)r_t\left\{\frac{Z_{t-1}^y}{P_t} + D_t^y + \frac{W_t}{P_t}N_t^y - C_t^y\right\}, \quad (13)$$

$$\frac{Z_t^y}{P_{t+1}} = \omega r_t\left\{\frac{Z_{t-1}^y}{P_t} + D_t^y + \frac{W_t}{P_t}N_t^y - C_t^y\right\}. \quad (14)$$

Note that the parameter γ disappears from the old's financial wealth dynamics. Although the surviving old's ex post rate of return on annuity is r_t/γ, only the fraction γ of the current old survive, and therefore the return on total financial wealth held by the old is given by r_t.

In sum, the dynamics of marginal propensity to consume for the old and for the young (equations 4 and 6), the dynamics of aggregate human and financial wealth for the old and for the young (equations 11, 12, 13, and 14), the aggregate consumption rule for them (equations 2 and 8), together with the value functions (equations 3 and 9) completely determine the aggregate dynamics on the part of the consumers. As is clear, the wealth distribution matters for the aggregate dynamics as well as the utilitarian welfare only through aggregate wealth for the two age groups.

2.2 Final goods producers

Final goods, Y_t, are produced by the final goods producers in a competitive market, who combine differentiated intermediate goods using the CES production function:

$$Y_t := \left[\int_0^1 (Y_{i,t})^{\frac{\kappa-1}{\kappa}}\, di\right]^{\frac{\kappa}{\kappa-1}}.$$

The parameter κ denotes the elasticity of substitution among differentiated intermediate goods. Given the aggregate price level, P_t, and the price of each intermediary goods, $P_{i,t}$, profit maximization implies the following iso-elastic demand for each intermediate good:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\kappa} Y_t. \quad (15)$$

2.3 Intermediate goods producers

Firm i in a monopolistically competitive market uses non-differentiated labor $L_{i,t}$ and capital $K_{i,t-1}$ in order to produce differentiated intermediate goods $Y_{i,t}$. The production function of the intermediate goods is given by

$$Y_{i,t} := L_{i,t}^{1-\alpha} K_{i,t-1}^{\alpha}, \tag{16}$$

where α is capital share. Labor is supplied by consumers with nominal wage rate W_t. Capital is rented to intermediary firms at real rate R_t^K from the capital producer. The real cost minimization problem is thus given by

$$\min \left(\frac{W_t}{P_t} L_{i,t} + r_t^K K_{i,t-1} \right)$$

subject to the production function (16). This gives the optimal factor price conditions:

$$\frac{W_t}{P_t} = (1-\alpha)\, \psi_t L_{i,t}^{-\alpha} K_{i,t-1}^{\alpha},$$

$$r_t^K = \alpha \psi_t L_{i,t}^{1-\alpha} K_{i,t-1}^{\alpha-1},$$

where ψ_t denotes real marginal costs.

Each firm operates in a monopolistically competitive market. In the baseline model, we assume that the intermediate goods producers face the nominal price rigidity due to a quadratic price adjustment costs (Rotemberg, 1982). Instantaneous real profit $\Pi_{i,t}^I$ is given by

$$\Pi_{i,t}^I := (1+\tau) \frac{P_{i,t}}{P_t} Y_{i,t} - \psi_t Y_{i,t} - \frac{\phi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t.$$

Here τ denotes a constant sales subsidy that potentially corrects the steady-state distortion stemming from monopolistic competition. This subsidy is financed by the lump sum tax to both types of consumers, but we set its baseline value to zero.[7]

We assume that the firms discount the one-period-ahead profit by the risk-free real interest rate, because there is no aggregate uncertainty, and that $m_{0,t} = 1/(r_1 \times r_2 \times \cdots \times r_t)$ acts as the pricing kernel. [8]

Hence, the profit maximization problem by price setting becomes

$$\max \sum_{t=0}^{\infty} m_{0,t} \Pi_{i,t}^I,$$

subject to the demand for intermediary goods in equation (15). Note that we have

[7]Note that even the lump sum tax is not neutral under heterogeneous consumers.

[8]In general, defining the pricing kernel in the heterogeneous agents economy is not trivial (see e.g. Carceles-Poveda and Coen-Pirani, 2009). As in Ghironi (2008) and Fujiwara and Teranishi (2008), we only conduct perfect foresight simulations, and therefore all assets yield same rates of return among different agents both *ex ante* and *ex post*, except for the very initial period in which a once-and-for-all, unexpected change in exogenous variables occurs.

implicitly assumed that the real interest rate is strictly positive, at least in the long run, because otherwise the above objective function is not well-defined.

Later we will focus on a symmetric equilibrium in which all intermediate goods producers set the same price, $P_{i.t} = P_t$ and $Y_{i,t} = Y_t$ for all i. In such an equilibrium, the instantaneous profit is also the same across firms and is given by:

$$\Pi^I_{i,t} = \Pi^I_t = \left[1 + \tau - \psi_t - \frac{\phi}{2}\pi_t^2\right] Y_t.$$

2.4 Capital producer

A capital producer purchases final goods, converts them into capital, and rents capital to intermediate goods producers. Its capital holding at the beginning of period $t+1$ is given by:

$$K_t = (1-\delta) K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{17}$$

where I_t denotes the amount of final goods used for investment and $(1-\delta) K_{t-1}$ is the end-of-period, after-depreciation capital in period t. The function $S(\cdot)$ denotes the investment growth adjustment costs used in Christiano et al. (2005):

$$S(x_t) := s\left(\frac{x_t^2}{2(1+n)^2} - \frac{x_t}{(1+n)} + \frac{1}{2}\right).$$

The capital producer maximizes the profit:

$$\sum_{t=0}^{\infty} m_{0,t} \Pi^K_t,$$

where the instantaneous profit is given by

$$\Pi^K_t := r^K_t K_{t-1} - I_t,$$

subject to the capital accumulation equation (17), taking the initial capital K_{-1} as given.

With q_t denoting the Lagrange multiplier on the period-t capital accumulation equation, the first-order conditions are given by:

$$1 = q_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \frac{1}{r_t} q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

and

$$q_t = \frac{1}{r_t}\left(r^K_{t+1} + (1-\delta)q_{t+1}\right).$$

The variable q_t is Tobin's (marginal) Q. The maximized ex-dividend value of the firm at the end of period t is denoted by Q^K_t and satisfies

$$Q^K_t = \frac{1}{r_t}\left(\Pi^K_{t+1} + Q^K_{t+1}\right).$$

The total share issued is normalized to and fixed at one, implying that Q^K_t equals the unit share price.

2.5 Mutual funds issuer

The mutual fund market is competitive. Each issuer is risk-neutral and discounts the future profits by the common pricing kernel $m_{0,t}$. Hence we can consider a representative issuer.

The representative issuer maximizes the value of mutual funds it issues. In each period, the issuer receives profits (dividends) from all intermediate goods producers, and buys and sells shares of all intermediate firms. It also purchases $e_{K,t}$ units of shares of the capital producers in period t. The representative issuer's period-t instantaneous profit is given by:

$$\begin{aligned}\Pi^M_t &= (\Pi^K_t + Q^K_t)e_{K,t-1} - Q^K_t e_{K,t} \\ &\quad + \int_0^1 (Q^I_{i,t} + D^I_{i,t})e_{i,t-1}di - \int_0^1 Q^I_{i,t}e_{i,t}di.\end{aligned}$$

Here, $e_{i,t}$ is the amount of share of the intermediate firm i purchased in period t, $Q^I_{i,t}$ is the real price of share of firm i, and $D^I_{i,t}$ is the real dividend payout per share from firm i. The issuer maximizes the sum of discounted profits:

$$\sum_{t=0}^{\infty} m_{0,t}\Pi^M_t.$$

To derive the optimality condition that is easily understandable, let us focus on a symmetric equilibrium in which all intermediate goods firms' profits and share prices are equalized, and in which all shares are held by the mutual fund issuer. In such an equilibrium, the instantaneous profit is given by:

$$\Pi^M_t = (\Pi^K_t + Q^K_t)e_{K,t-1} - Q^K_t e_K + (Q^I_t + D^I_t)\int_0^1 e_{i,t-1}di - Q^I_t\int_0^1 e_{i,t}di.$$

The solution to the issuer's problem has to be interior and to satisfy the first-order condition for $e_{i,t}$ and for $e_{K,t}$:

$$-m_{0,t}Q_t^I + m_{0,t+1}(Q_{t+1}^I + D_{t+1}^I) = 0,$$

and

$$-m_{0,t}Q_t^K + m_{0,t+1}(Q_{t+1}^K + \Pi_{t+1}^K) = 0,$$

Because $m_{0,t+1}/m_{0,t} = 1/r_t$, we have:

$$Q_t^I = \frac{1}{r_t}(Q_{t+1}^I + D_{t+1}^I)$$

and

$$Q_t^K = \frac{1}{r_t}(Q_{t+1}^K + \Pi_{t+1}^K)$$

Normalizing the total shares issued by a firm to one, $D_t^I = \Pi_t^I$ and Q_t^I equals the value of an intermediate goods producer. The instantaneous profit in equilibrium is

$$\Pi_t^M = (\Pi_t^K + Q_t^K) - Q_t^K + (Q_t^I + D_t^I) - Q_t^I = \Pi_t^K + D_t^I$$

Hence, the end-of-period value of the mutual funds in period t is the sum of Q_t^K and Q_t^I, and satisfies the following recursion:

$$Q_t^K + Q_t^I = \frac{1}{r_t}\left\{\Pi_{t+1}^M + Q_{t+1}^K + Q_{t+1}^I\right\}.$$

2.6 Monetary policy

The central bank adopts strict inflation targeting so as to keep inflation at a constant level, $\pi_t = \overline{\pi}$ for all t.

2.7 Market clearing conditions

The financial market clears with

$$Q_t^K + Q_t^I = \frac{(Z_t^y + Z_t^o)/P_t}{R_t},$$

which equates the end-of-period total value of mutual funds on the left-hand side to the total real value of mutual fund demand on the right-hand side. On the right-hand side, both Z_t^y and Z_t^o are divided by the price P_t and by R_t, because they are nominal values and because they include the nominal interest rate.

The labor market clears when the labor demand equals the young's population:

$$\int_0^1 L_{i,t} di = N_t^y,$$

and the rental market for capital clears if

$$\int_0^1 K_{i,t-1} di = K_{t-1}.$$

The good market clears as

$$Y_t = C_t + I_t + \frac{\phi}{2} (\pi_t - 1)^2 Y_t,$$

where $C_t = C_t^y + C_t^o$.

The sales subsidy paid to intermediate goods producers is financed through the lump-sum tax:

$$\frac{D_t^o}{N_t^o} = \frac{D_t^y}{N_t^y} = \frac{-\tau Y_t}{N_t}.$$

In numerical computation, we use a de-trended version of the model. The detail of de-trending is described in Appendix B.

3 Optimal inflation rates in the long run

In this section, we calibrate the model and compute the welfare-maximizing inflation rates in the long-run for the young and the old under the assumption that surprise inflation does not cause wealth redistribution in the initial period. We also explore how optimal inflation rates change with demographic structure, by changing parameters such as γ and b.

3.1 Calibration

The parameter calibration is shown in Table 1. The model is simulated at a quarterly frequency. The discount factor β and capital depreciation δ are set at $1.04^{-1/4}$ and $1.01^{-1/4} - 1$, respectively. Under our benchmark calibration, we set the parameters for demographic dynamics ω and γ so that on average, each individual works for 45 years and lives as an old agent for 15 years. They are set to $(45 \times 4 - 1)/(45 \times 4) = 0.9944$

and $(15 \times 4 - 1)/(15 \times 4) = 0.9833$. Population growth rate is set to zero, which implies $b = 1 - \omega = 0.0055$. Other parameters are set to conventional values following Fujiwara and Teranishi (2008). Capital share α and elasticity of substitution of intermediate goods κ are set to $1/3$ and 10, respectively. For the parameter of Rotemberg (1982) cost ϕ, we use 50 so that the New Keynesian Philips Curve of our model matches with the one implied by Calvo (1983) price setting where one forth of firms change prices in each period on average. Parameter defining investment adjustment costs s is set 2.48, which is taken from Christiano et al. (2005). Elasticity of intertemporal substitution σ is set to 0.5 which is consistent with Yogo (2004). Also, for the benchmark case, τ is set to zero.

Table 1: Benchmark Parameter Values

	Parameters	Values
ω	transition probability to old	0.9944
γ	survival rate	0.9833
b	birth rate	$1 - \omega = 0.0055$
β	discount factor	$1.04^{-\frac{1}{4}}$
σ	IES	0.5
ρ	Curvature	$\frac{\sigma-1}{\sigma} = -1$
α	capital share	$\frac{1}{3}$
κ	elasticity of substitution	10
ϕ	Rotemberg cost parameter	50
δ	capital depreciation rate	$1.01^{\frac{1}{4}} - 1$
s	investment adjustment costs parameter	2.48

3.2 Within-group utilitarian welfare

As a welfare metric, we use within-group utilitarian welfare in period 0. Here, the assumption of RINCE preferences *à la* Farmer (1990) is helpful, because it enables us to derive the closed form solutions for utilitarian welfare for groups of the young and the old. This greatly simplifies the analysis in this paper and contributes to offering a more intuitive explanation of the non-zero optimal inflation rates.

Because the value functions for the young and for the old are linear in total wealth, within-group utilitarian welfare can be obtained by linearly aggregating them. After de-trending by the young's population, N_t^y, welfare for the young and the old at time t takes the following form:

$$v_t^y = \left(mpc_t^y\right)^{\frac{\rho-1}{\rho}} \left(\frac{r_{t-1}}{1+n} \frac{a_{t-1}^y}{P_{t-1}} + h_t^y \right), \tag{18}$$

and

$$v_t^o = (mpc_t^o)^{\frac{\rho-1}{\rho}} \left(\frac{r_{t-1}}{1+n} \frac{a_{t-1}^o}{P_{t-1}} + h_t^o \right), \tag{19}$$

where variables in the lowercase letters, $(a_{t-1}^y, a_{t-1}^o, h_t^y, h_t^o, v_t^y, v_t^o)$, stand for de-trended variables so that $x_t := X_t / N_t^y$.

The welfare measure we use is those evaluated at the beginning of transition from period 0: v_0^o and v_0^y. We investigate, starting from the zero-inflation steady state, what values of new $\overline{\pi}$ are most preferred by the period-0 young and by the period-0 old. We call these rates *the optimal inflation rates for the young and for the old*, respectively. Throughout the paper, initial states are set to their values in the zero-inflation steady state.

3.3 Why do the young and the old have different preferences over inflation?

Before showing the results, let us briefly discuss the potential sources of different inflation preferences of the young and the old.

Earning heterogeneity First, the young and the old may have different preferences over the long-run inflation because inflation affects their income streams differently. Recall that the young receives labor income whereas the old do not. Because of the Phillips curve, higher inflation is associated with higher marginal cost of production. Higher marginal cost raises the real wage and the capital rental rate, but at the same time suppresses the intermediate goods producers' profits. There are counteracting forces to the value of mutual funds — a rise in the capital producer's value and a decline in the intermediate goods producers' value — and the old are affected by these forces. In addition to these two counteracting forces, the young benefit also from labor compensation that increases with inflation. Therefore, as far as the earning redistribution effect of inflation is concerned, the young tend to prefer higher inflation than do the old.

Redistribution through the initial nominal asset holdings An unanticipated, permanent change of inflation target that occurs in period 0 typically have some real, redistributional consequences through the initial holdings of nominal assets, because their values (and hence their real returns) unexpectedly change in response to a surprise change in the period-0 price level. However, in the present setting, such redistributional effects are absent due to the Modigliani-Miller theorem. Therefore, we do not need to specify how much of the initial assets issued by firms are nominal or real.

To see this, consider a surprise increase in the period-0 price level. It reduces the real value of nominal liabilities of firms, but increases the real value of their equity by the same amount because firms' profits increases as much as the reduced debt repayment. Because the representative mutual fund issuer holds all assets issued by both final and intermediate goods producers and by the representative capital producer, the mutual funds' value is unaffected by a surprise inflation that occurs in period 0. Because the households own firms only through the mutual funds, their optimization problems are also unaffected.

Therefore, the main source of the preference heterogeneity between the young and the old is the earning heterogeneity channel.

3.4 Welfare-maximizing inflation targets: the representative young and old households

Figure 1 shows the representative old's (the left panel) and young's preferences (the right panel) over inflation targets. On the horizontal axis is the target rate of inflation and both the associated time-0 and steady-state welfare numbers are depicted.

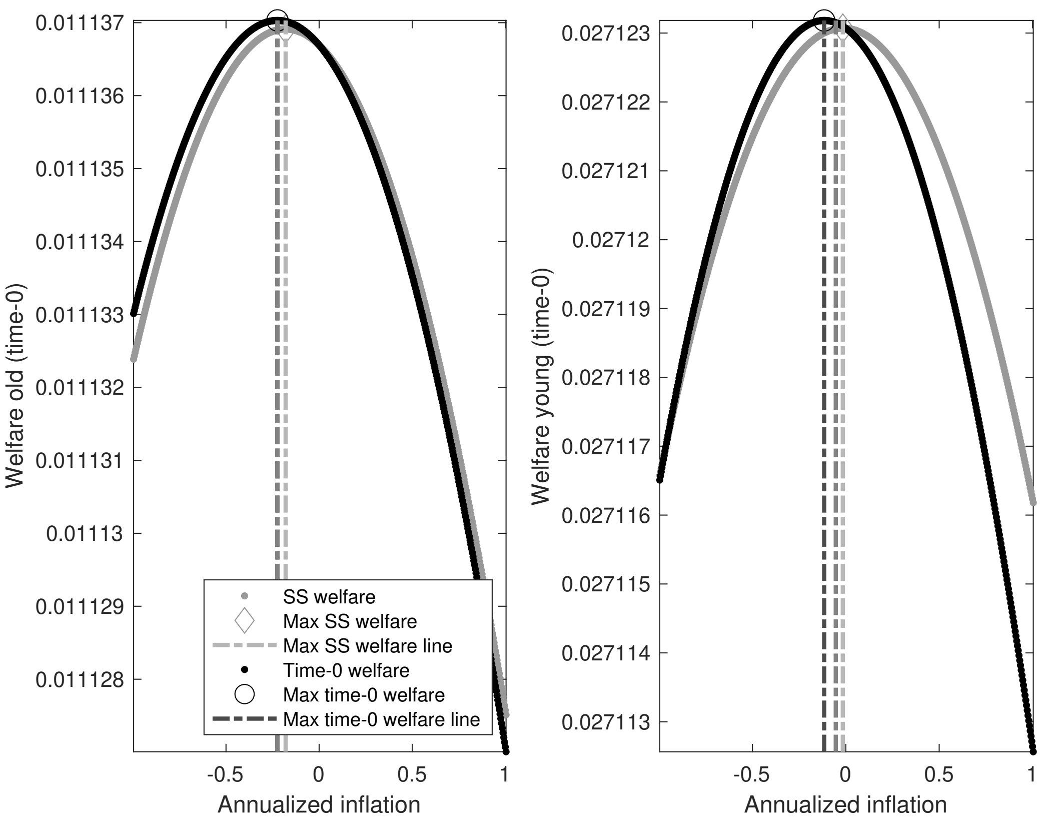

Figure 1: Welfare-maximizing inflation targets

Let us first look at the steady-state welfare-maximizing inflation targets. Zero inflation target approximately maximizes the representative young household's steady-state welfare, whereas the representative old household's steady-state welfare is maximized at a negative value of inflation. The difference between the representative old and young households' preferences is mainly due to earning heterogeneity: higher inflation is accompanied by higher marginal costs and earning redistribution occurs from asset holders to workers. Because the old households are asset holders and do not earn labor income, they prefer deflation that redistributes earning toward them. In contrast, the young households earn labor income but they also hold assets. Hence, the overall redistribution effect of inflation is almost zero for the representative young household.

What about the time-0 welfare-maximizing inflation targets? These targets are lower than their steady-state welfare-maximizing counterparts both for the old and for the young households, but the difference is bigger for the young households. Why do they prefer lower inflation target? The reason is that lower inflation is associated with a lower level of steady-state capital (as inflation is not too low), and that both the representative young and old households who are alive in period 0 can entertain higher consumption along the transition.

Still, the representative old household prefers lower inflation than the representative young household, and thus the population-weighted average of their preferred inflation target will be affected by population aging.

3.5 Welfare-maximizing inflation targets: a further decomposition

In our model, households are heterogeneous even within the same age group. Even in the steady state, young households differ in their financial wealth holdings because some have been young for a long period of time without being hit by a probabilistic aging shock, and because some are relatively newly born and have not yet accumulated much financial wealth. Old households also differ in their financial wealth: they might have become old with different wealth levels, and they also differ in the number of periods from when they became old.

In Figure 2, we plot, separately for the old and young households, the most preferred rates of inflation target for different levels of financial wealth. The cumulative distribution functions of financial wealth in period 0 (i.e., in the zero-inflation steady state) are also displayed respectively for the young and the old households.

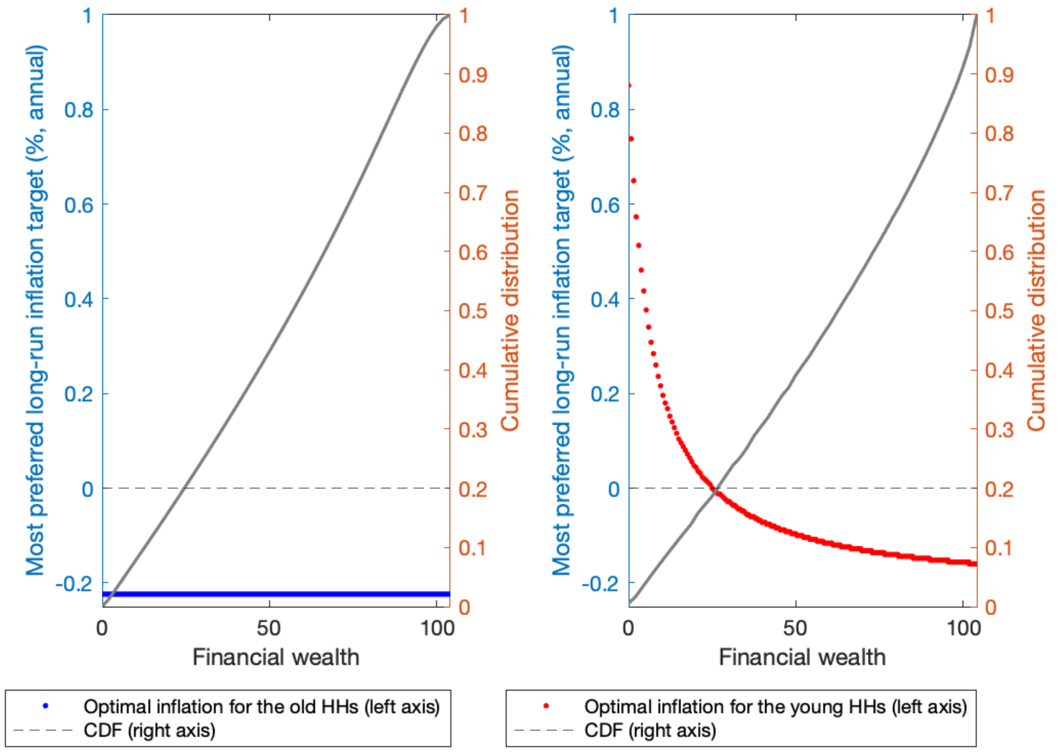

Figure 2: Welfare-maximizing inflation targets

In the left panel, it is clear that the old households' preferences are perfectly aligned and their most preferred inflation target is independent of their financial wealth. This is because their human wealth is zero and, therefore, their maximized utility is linear in their time-0 financial wealth.

In contrast, the young households prefer lower inflation target as their financial wealth increase, as shown in the right panel. Young households who are relatively newly born have little to no financial wealth, and human wealth is relatively more important for them than financial wealth. Hence, they prefer high inflation that redistributes from asset holders to workers. However, young households keep accumulating financial wealth unless they are hit by an aging shock, and their preferred inflation target keeps declining as they stay young for longer. Because young households quickly accumulate financial wealth, only about 20 percent of young households prefer positive inflation target and the remaining 80 percent prefer deflation.

In sum, there is no disagreement within the old households about a desirable inflation target, whereas young households' preferences are diverse.

3.6 Intergenerational agreement about inflation target

We also examine whether the representative young and old households in different time periods have different preferences over the long-run inflation target. Examining it is important because it would be unrealistic to assume that a constant inflation target is achieved if there are large disagreement across generations.

Perhaps surprisingly, the representative young and old households in different time periods broadly agree about what a desirable rate of inflation target is. In Figure 3, we plot the lifetime utility of the representative old (left panel) and young (right panel) households, respectively, in different time periods. Although the utility levels are different across time periods, they are maximized at the same rate of inflation target. In other words, if we ask future representative young and old households what constant rate of inflation target they would like to achieve from that period onward, they would give consistent answers.

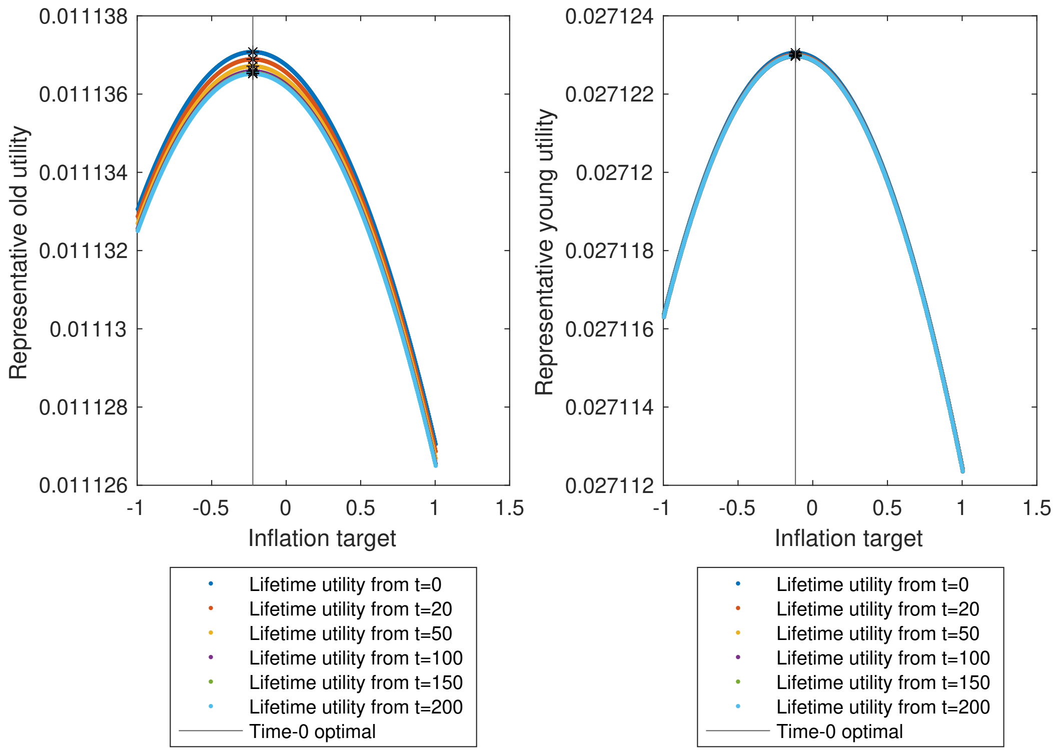

Figure 3: Welfare and welfare maximizing inflation rate in different periods

4 Does population aging strengthen societal preferences for deflation?

Having established that young and old households have different preferences for a long-run inflation target, now we turn to whether and how population aging affects socially preferred target rate of inflation. Two sources of population aging are examined: longer life expectancy and a lower birth rate. Although the composition effect acts to reduce the socially preferred target rate of inflation, population aging may bring about other changes that affect the welfare-maximizing inflation targets for young and old households. In this section we begin with a numerical analysis to show that the overall effect of aging on the population-weighted optimal inflation rates is quantitatively small in the steady state and that it is indeed positive if the transition is taken into account. Then we move on to a theoretical steady-state analysis to reveal that the preferences of young and old households becomes more similar under greater life expectancy and a lower birth rate, either through a direct effect on the marginal propensities to consume or an indirect effect through a lower real interest rate.

4.1 Life Expectancy

In Figure 4 we plot how the life expectancy affects the rates of inflation that maximize the representative young and old's steady-state welfare. The vertical axis shows the optimal annualized inflation rate and the horizontal axis shows life expectancy for the old defined by γ.

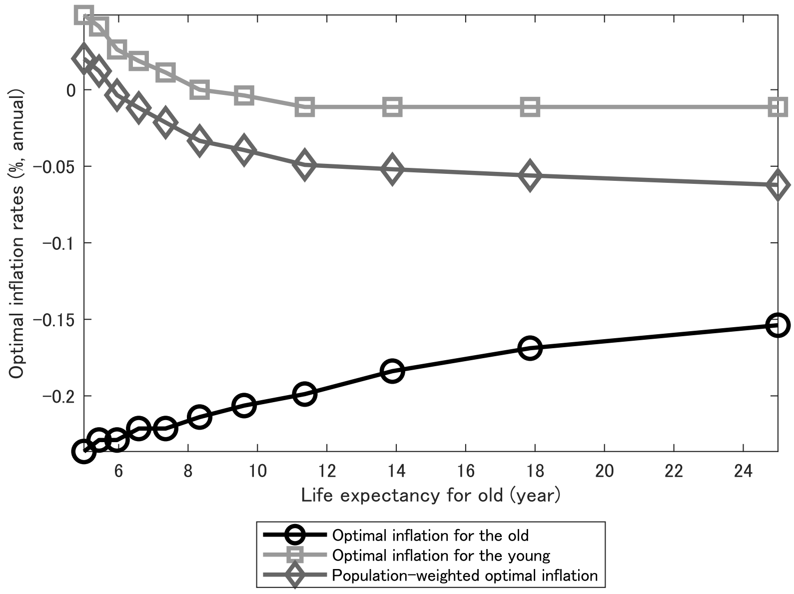

Figure 4: The steady-state welfare maximizing inflation rate by life expectancy

The steady-state welfare maximizing inflation is declining for the young households but is increasing for the old households as life expectancy becomes longer. With greater life expectancy, both the young and the old households effectively become more patient. Therefore, their steady-state financial wealth increase and the steady-state real interest rate declines.

For the young households, a lower real interest rate acts to increase their human wealth through a lower discount rate and higher real wage. However, under our calibration of intertemporal elasticity of substitution being smaller than unity, a lower real interest rate incentivizes them to increase their saving, leading to lower steady-state consumption and higher financial wealth. As the young households hold more financial wealth relative to human wealth, inflation becomes more costly for them. This is why the steady-state welfare maximizing inflation is declining for the young households as life expectancy increases.

For the old households, who receive no labor income, increased financial wealth make them prefer lower inflation through an earnings redistribution channel. However, in response to a lower real rate, the old households increase their savings by cutting back on their consumption, which is undesirable. Here the latter negative effect of deflation dominates the former. Therefore, when subject to a downward pressure on the real rate caused by increased life expectancy, the old households' preferences for deflation is somewhat mitigated.

What about the time-0 welfare maximizing inflation? These rates are increasing in the life expectancy. Figure 5 plots how the life expectancy affects the rates of inflation that maximize the representative young and old households' time-0 welfare. When calculating these numbers, the initial condition for each value of γ is different and set to the associated zero-inflation steady state. Both the representative young and old households prefer higher inflation target as the life expectancy becomes longer, and, as a result, the population-weighted average of their most preferred targets also increases with the life expectancy.

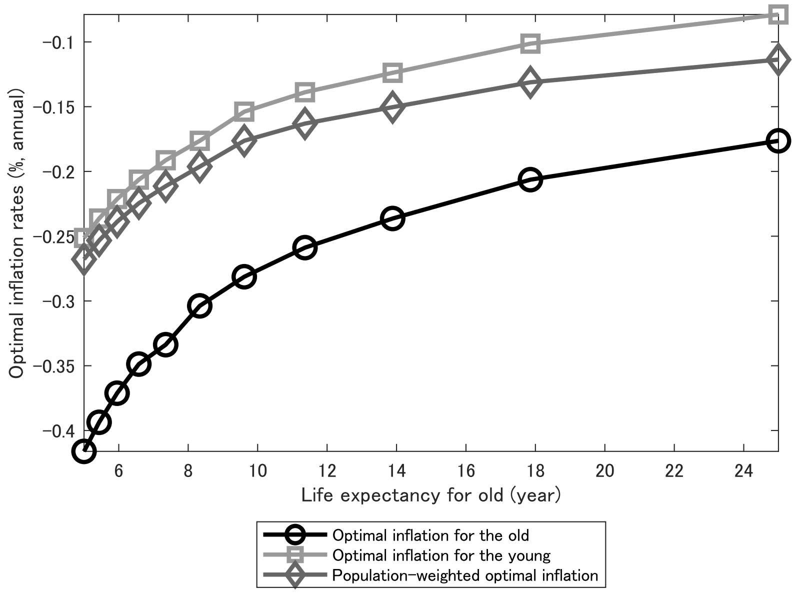

Figure 5: The time-0 welfare maximizing inflation rate by life expectancy

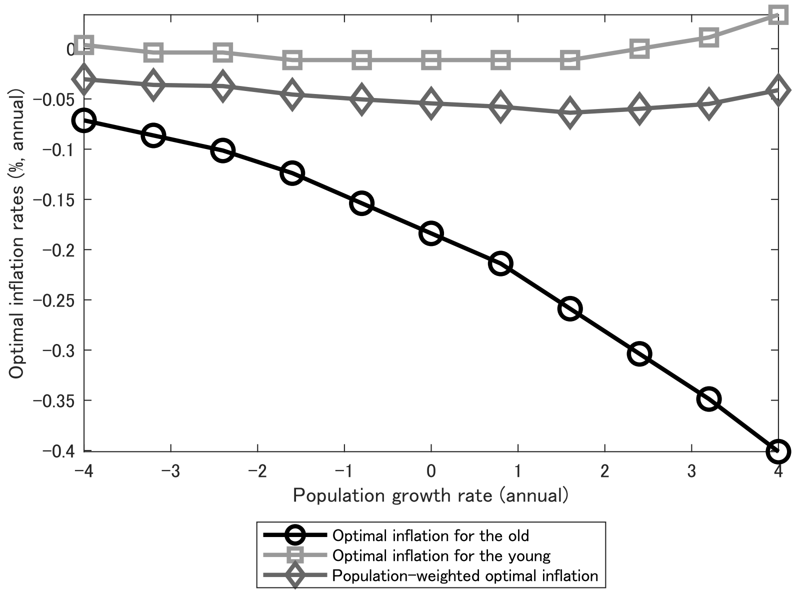

Figure 6: The steady-state welfare maximizing inflation rate by population growth rate

As we saw already, the representative young household's time-0 welfare is maximized at a negative rate of inflation under the baseline parameterization. This property is unchanged for different values of γ. It is however increasing with the life expectancy parameter γ, unlike the steady-state welfare maximizing inflation rate, which is decreasing. As a result, both the representative young and old households prefer higher inflation in an economy with longer life expectancy.

Therefore, when population aging is caused by increased life expectancy, society prefers less deflation.

4.2 Population Growth

Figure 6 depicts the effect of the birth rate on the steady-state welfare maximizing inflation rates. Because the population growth rate varies as we vary the birth rate, we put the population growth rate on the horizontal axis. As the birth rate increases, the steady-state welfare maximizing inflation target for the representative old household declines, whereas that for the representative young household is approximately constant around zero and increases by a little when the population growth rate is sufficiently increased.

Figure 7 plots how the population growth rate affects the rates of inflation that maximize the representative young and old households' time-0 welfare. Here, both inflation rates are negative and decline as the population growth rate is increased. In other words, when population aging is caused by a decline in the birth rate, population growth rate declines and society prefers less deflation.

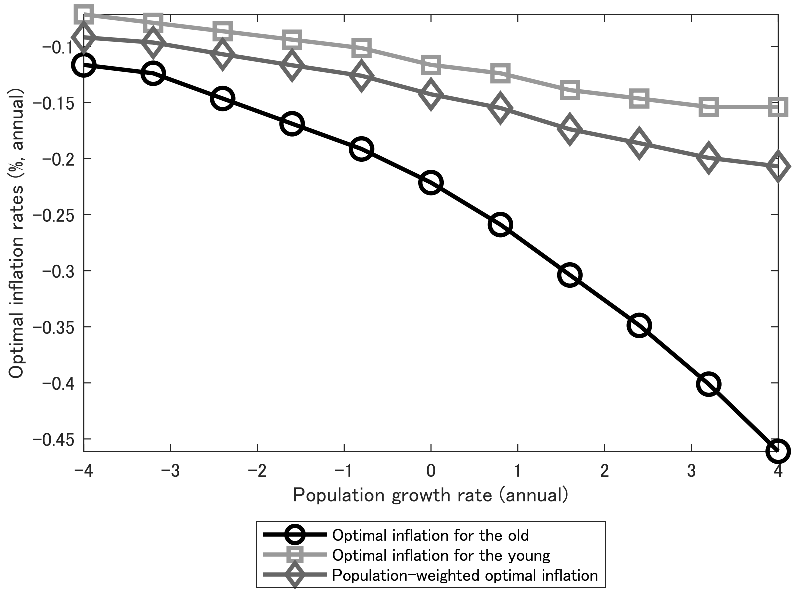

Figure 7: The time-0 welfare maximizing inflation rate by population growth rate

4.3 Understanding the mechanism

Tractability of our model helps us understand why population aging increases the population-weighted optimal inflation target rates, despite the negative composition effect.

Equations (18) and (19) reveal that the marginal propensity to consume (MPC) and financial and human wealth determine the welfare for young and old households. The MPC's for young and old households in the steady state are given by:

$$mpc^o = 1 - \beta^{\frac{1}{1-\rho}} \gamma r^{\frac{\rho}{1-\rho}}, \tag{20}$$

$$mpc^y = 1 - \beta^{-\frac{1}{\rho-1}} \left\{ \omega + (1-\omega) \left(\frac{mpc^o}{mpc^y} \right)^{\frac{\rho-1}{\rho}} \right\}^{-\frac{\rho}{\rho-1}} r^{-\frac{\rho}{\rho-1}}. \tag{21}$$

The following proposition summarizes how the MPC's respond to changes in longevity and the real interest rate. The proof is in Appendix C.

Proposition 1. *Suppose $\rho < 0$. Then the following properties hold in the steady state: (1) the old households' MPC out of wealth, mpc^o, is unambiguously decreasing in γ and increasing in the real interest rate r; (2) given everything else equal, when the old households' MPC changes, the young households' MPC changes in the same direction but less than one-for-one with it, and mpc^y is increasing in the real interest rate; finally, (3) the relative MPC, mpc^o/mpc^y, is bigger than one and is decreasing both in γ and r.*

Let us discuss what happens in response to population aging. Holding the real interest rate fixed, greater longevity (higher γ) reduces the MPC's for young and old households, and the effect is larger for old households so that the relative MPC's becomes closer to one. Hence, the MPC's in equations (18) and (19) become closer to each other, making the preferences of young and old households more aligned. Because all households consume less and save more, both young and old households possess more financial wealth. Because young households have more financial wealth, the redistribution effect of inflation from asset income to labor income becomes less desirable for young households, making them prefer lower inflation. Old households benefit less from deflation, because a larger fraction of redistributional gain now goes to young households, and prefer higher inflation. At the same time, because all households accumulate more wealth, the real interest rate declines, and a lower real interest rate reduces the MPC's, reiterating the effect of higher γ.

The effect of a lower birth rate is indirect and operates through a lower interest rate. A lower birth rate implies lower labor supply, increasing the real wage, and the young households accumulate savings more rapidly. Larger financial wealth held by young households then lower the real interest rate, reducing the MPC's for young and old households, making them consume less and save more and lowering the real rate further. Because young households have larger financial wealth, they prefer lower inflation. Old households prefer higher inflation, because the gain from redistribution through lower inflation becomes smaller for them.

5 Conclusion

This paper examines whether and how population aging affects societal preferences for the long-run inflation target, using an overlapping-generations New Keynesian model as a laboratory. In our model, young and old households are differentially affected by the target inflation rate set by the central bank, through an earning redistribution channel: inflation increases the real wage but reduces the firms' profits that are distributed to asset holders. This redistribution effect of inflation target results in heterogeneous preferences of households over the long-run inflation target.

When the steady-state welfare is concerned, the welfare-maximizing inflation target for the representative old household is negative, whereas that for the representative young household is close to zero. This difference is largely explained by the earning redistribution channel. In contrast, if the welfare for the households that are present in the initial period is concerned, the welfare-maximizing inflation targets are negative for both the representative young and old households, with that for the old household being lower than that for the young household. The households that are present in the initial period benefit from higher consumption along the transition to a steady state with lower capital, at the expense of future newborns who experience lower real wages.

However, population aging, whether it is due to increased life expectancy or a lower birth rate, acts to increase the time-0 welfare-maximizing inflation targets for both the representative young and old households in the model. As a result, in contrast to Bullard et al. (2012), the earning redistribution channel *per se* does not produce a negative link between population aging and a socially preferred rate of inflation target. Moreover, the quantitative contribution of the channel is limited. An investigation of whether there are mechanisms that give rise to such a relationship in our model is left for future studies.

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Appendix

A Verifying the optimality of the value function and the decision rule

A.1 The old

For any t, using the period-t budget constraint and the recursion for H_t^o, we obtain:

$$\begin{aligned}\frac{Z_t^o}{P_{t+1}} + H_{t+1}^o &= \frac{r_t}{\gamma}\left\{\frac{Z_{t-1}^o}{P_t} + D_t^o - C_t^o\right\} + H_{t+1}^o \\ &= \frac{r_t}{\gamma}\left\{\frac{Z_{t-1}^o}{P_t} + D_t^o - C_t^o\right\} + \frac{r_t}{\gamma}\left\{H_t^o - D_t^o\right\} \\ &= \frac{r_t}{\gamma}\left\{\frac{Z_{t-1}^o}{P_t} + H_t^o - C_t^o\right\}.\end{aligned}$$

Consider maximizing

$$\left\{(C_t^o)^\rho + \beta\gamma\left(V_{t+1}^o\right)^\rho\right\}^{\frac{1}{\rho}}$$

subject to the above equation and the value function (3),

$$V_{t+1}^o = \left(mpc_{t+1}^o\right)^{\frac{\rho-1}{\rho}} \times \left(Z_t^o/P_{t+1} + H_{t+1}^o\right).$$

Once the two constraints are substituted into the objective function to eliminate future wealth, the problem reduces to the unconstrained problem where the objective function is given by:

$$\left\{(C_t^o)^\rho + \beta\gamma^{1-\rho}r_t^\rho(mpc_{t+1}^o)^{\rho-1}\left(\frac{Z_{t-1}^o}{P_t} + H_t^o - C_t^o\right)^\rho\right\}^{\frac{1}{\rho}}.$$

Taking the first-order condition in C_t^o, we obtain:

$$(C_t^o)^{\rho-1} = \beta\gamma^{1-\rho}r_t^\rho(mpc_{t+1}^o)^{\rho-1}\left(\frac{Z_{t-1}^o}{P_t} + H_t^o - C_t^o\right)^{\rho-1},$$

and hence

$$\left(\frac{\frac{C_t^o}{Z_{t-1}^o/P_t + H_t^o}}{1 - \frac{C_t^o}{Z_{t-1}^o/P_t + H_t^o}}\right)^{\rho-1} = \beta\gamma^{1-\rho}r_t^\rho \times \left(mpc_{t+1}^o\right)^{\rho-1}.$$

By comparing this equation with the recursion for $\{mpc_t^o\}$, it is clear that the ratio of the optimal consumption to wealth in t must be equal to mpc_t^o.

With $C_t^o = mpc_t^o \times (Z_{t-1}^o/P_t + H_t^o)$, the value function in t can be calculated as follows:

$$\begin{aligned}
V_t^o &= \left\{(C_t^o)^\rho + \beta\gamma^{1-\rho} r_t^\rho (mpc_{t+1}^o)^{\rho-1}\left(\frac{Z_{t-1}^o}{P_t} + H_t^o - C_t^o\right)^\rho\right\}^{\frac{1}{\rho}} \\
&= \left(\frac{Z_{t-1}^o}{P_t} + H_t^o\right) \times \left\{\left(\frac{C_t^o}{\frac{Z_{t-1}^o}{P_t} + H_t^o}\right)^\rho + \beta\gamma^{1-\rho} r_t^\rho (mpc_{t+1}^o)^{\rho-1}\left(\frac{\frac{Z_{t-1}^o}{P_t} + H_t^o - C_t^o}{\frac{Z_{t-1}^o}{P_t} + H_t^o}\right)^\rho\right\}^{\frac{1}{\rho}} \\
&= \left(\frac{Z_{t-1}^o}{P_t} + H_t^o\right) \times \left\{(mpc_t^o)^\rho + \beta\gamma^{1-\rho} r_t^\rho (mpc_{t+1}^o)^{\rho-1}(1 - mpc_t^o)^\rho\right\}^{\frac{1}{\rho}} \\
&= \left(\frac{Z_{t-1}^o}{P_t} + H_t^o\right) \times \left\{(mpc_t^o)^\rho + \left(\frac{mpc_t^o}{1 - mpc_t^o}\right)^{\rho-1}(1 - mpc_t^o)^\rho\right\}^{\frac{1}{\rho}} \\
&= \left(\frac{Z_{t-1}^o}{P_t} + H_t^o\right) \times \left\{(mpc_t^o)^\rho + (mpc_t^o)^{\rho-1}(1 - mpc_t^o)\right\}^{\frac{1}{\rho}} \\
&= \left(\frac{Z_{t-1}^o}{P_t} + H_t^o\right) \times \left\{(mpc_t^o)^{\rho-1}\right\}^{\frac{1}{\rho}} \\
&= \left(\frac{Z_{t-1}^o}{P_t} + H_t^o\right) \times (mpc_t^o)^{\frac{\rho-1}{\rho}}.
\end{aligned}$$

Hence the value function and the decision rules specified in equations (2) to (3) solves the dynamic programming problem for the old.

A.2 The young

Consider a young agent in period t with the beginning-of-period-t nominal asset being Z_{t-1}^y. After consuming C_t^y, her beginning-of-period-$(t+1)$ real asset is equal to the right hand side of equation (10), i.e.

$$r_t\left\{\frac{Z_{t-1}^y}{P_t} + D_t^y + \frac{W_t}{P_t} - C_t^y\right\}.$$

Hence, if she becomes old in period $t+1$, her wealth at the beginning of $t+1$ equals

$$r_t\left\{\frac{Z_{t-1}^y}{P_t} + D_t^y + \frac{W_t}{P_t} - C_t^y\right\} + H_{t+1}^o,$$

whereas if she stays young in period $t+1$, her wealth at the beginning of $t+1$ equals

$$r_t \left\{ \frac{Z^y_{t-1}}{P_t} + D^y_t + \frac{W_t}{P_t} - C^y_t \right\} + H^y_{t+1}.$$

Using the value function formulas (equations 3 and 9), the expected next period value is given by

$$\begin{aligned}
\omega V^y_{t+1} + (1-\omega)\, V^o_{t+1} &= \omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} \times \left(r_t \left\{ \frac{Z^y_{t-1}}{P_t} + D^y_t + \frac{W_t}{P_t} - C^y_t \right\} + H^y_{t+1} \right) \\
&\quad +(1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} \times \left(r_t \left\{ \frac{Z^y_{t-1}}{P_t} + D^y_t + \frac{W_t}{P_t} - C^y_t \right\} + H^o_{t+1} \right) \\
&= \left(\omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} \right) r_t \left\{ \frac{Z^y_{t-1}}{P_t} + D^y_t + \frac{W_t}{P_t} - C^y_t \right\} \\
&\quad + \left(\omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} H^y_{t+1} + (1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} H^o_{t+1} \right) \\
&= \left(\omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} \right) \\
&\quad \times \left[r_t \left\{ \frac{Z^y_{t-1}}{P_t} + D^y_t + \frac{W_t}{P_t} - C^y_t \right\} + r_t \left\{ H^y_t - \left(D^y_t + \frac{W_t}{P_t} \right) \right\} \right] \\
&= \left(\omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} \right) \times r_t \left\{ \frac{Z^y_{t-1}}{P_t} + H^y_t - C^y_t \right\}.
\end{aligned}$$

Therefore, her objective function in period t can be written as

$$\left[(C^y_t)^\rho + \beta \left(\omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} \right)^\rho \times r^\rho_t \left\{ \frac{Z^y_{t-1}}{P_t} + H^y_t - C^y_t \right\}^\rho \right].$$

Taking the first-order condition, we obtain:

$$(C^y_t)^{\rho-1} = \beta r^\rho_t \left(\omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} \right)^\rho \left(\frac{Z^y_{t-1}}{P_t} + H^y_t - C^y_t \right)^{\rho-1},$$

and hence

$$\left(\frac{\frac{C^y_t}{Z^y_{t-1}/P_t + H^y_t}}{1 - \frac{C^y_t}{Z^y_{t-1}/P_t + H^y_t}} \right)^{\rho-1} = \beta r^\rho_t \times \left(\omega (mpc^y_{t+1})^{\frac{\rho-1}{\rho}} + (1-\omega)(mpc^o_{t+1})^{\frac{\rho-1}{\rho}} \right)^\rho.$$

By comparing this equation with the recursion for $\{mpc^y_t\}$, it is clear that the ratio of the optimal consumption to wealth in t must be equal to mpc_t^y. Checking the value function formula is straightforward and the same as the old's, and therefore is omitted.

B De-trended equilibrium conditions

For computation, we de-trend all growing variables by the young's population, N_t^y, and denote the de-trended variables by their corresponding lowercase letters. To be consistent with the notations used in Gertler (1999) and Fujiwara and Teranishi (2008), we denote the young's marginal propensity to consume out of wealth by $\theta_t := mpc_t^y$, and the old's relative marginal propensity by $\epsilon_t := mpc_t^o / mpc_t^y$, and also introduce an auxiliary variable:

$$\Phi_t := \omega + (1-\omega)\,\epsilon_t^{\frac{\rho-1}{\rho}}.$$

Let $A_t^y := Z_t^y / R_t$ and $A_t^o := Z_t^o / R_t$.

The system of equations other than the monetary policy rule is given as follows. The production function is written as

$$y_t = \left(\frac{k_{t-1}}{1+n}\right)^{\alpha}.$$

The real wage and the capital rental rate equal to the marginal products of labor and capital, respectively:

$$\frac{W_t}{P_t} = (1-\alpha)\,\psi_t \left(\frac{k_{t-1}}{1+n}\right)^{\alpha},$$

and

$$r_t^K = \alpha\psi_t \left(\frac{k_{t-1}}{1+n}\right)^{\alpha-1}.$$

Flow income other than labor income consist of the lump-sum transfer:

$$d_t^y = -\frac{1}{1+\Gamma}\tau y_t, \tag{22}$$

$$d_t^o = -\tau\frac{\Gamma}{1+\Gamma} y_t.$$

The Fisher equation is given by:

$$r_t = R_t / \pi_{t+1},$$

where π_t denotes the gross inflation rate:

$$\pi_t := \frac{P_t}{P_{t-1}}.$$

The New Keynesian Phillips curve is:

$$-\phi\left(\pi_t - 1\right)\pi_t y_t + \left(\psi_t - 1\right)\kappa y_t + \frac{1+n}{r_t}\phi\left(\pi_{t+1} - 1\right)\pi_{t+1} y_{t+1} = 0. \tag{23}$$

The capital accumulation equation and the capital producer's first-order condition are given by:

$$k_t = (1-\delta)\frac{k_{t-1}}{1+n} + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t,$$

$$1 = q_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right] + \frac{1}{r_t} q_{t+1} S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2.$$

The value of intermediate goods producers and of the capital producers are given by:

$$q_t^I = \frac{(1+n)}{r_t}\left[q_{t+1}^I + \left(1 + \tau - \psi_{t+1} - \frac{\phi}{2}(\pi_{t+1} - 1)^2\right) y_{t+1}\right].$$

$$q_t = \frac{1}{r_t}\left[q_{t+1}\left(1-\delta\right) + r_{t+1}^K\right].$$

The asset market clearing condition is

$$q_t^K + q_t^I = \frac{a_t^y + a_t^o}{P_t}.$$

The old households' optimal decision rule is given by:

$$c_t^o = \epsilon_t \theta_t\left(\frac{r_{t-1}}{1+n}\frac{a_{t-1}^o}{P_{t-1}} + h_t^o\right), \tag{24}$$

$$\left(\frac{\epsilon_t \theta_t}{1 - \epsilon_t \theta_t}\right)^{\rho - 1} = \beta\gamma^{1-\rho} r_t^\rho \left(\epsilon_{t+1}\theta_{t+1}\right)^{\rho - 1},$$

and the total asset held by the old households evolve as:

$$\frac{a_t^o}{P_t} = \frac{r_{t-1}}{1+n}\frac{a_{t-1}^o}{P_{t-1}} - c_t^o + d_t^o + (1-\omega)\left(\frac{r_{t-1}}{1+n}\frac{a_{t-1}^y}{P_{t-1}} + \frac{W_t}{P_t} - c_t^y + d_t^y\right).$$

We have a set of similar conditions for the young households:

$$c_t^y = \theta_t \left(\frac{r_{t-1}}{1+n} \frac{a_{t-1}^y}{P_{t-1}} + h_t^y \right), \tag{25}$$

$$\left(\frac{\theta_t}{1-\theta_t} \right)^{\rho-1} = \beta \left(r_t \Phi_{t+1} \right)^{\rho} \left(\theta_{t+1} \right)^{\rho-1},$$

$$\frac{a_t^y}{P_t} = \omega \left(\frac{r_{t-1}}{1+n} \frac{a_{t-1}^y}{P_{t-1}} + \frac{W_t}{P_t} - c_t^y + d_t^y \right)$$

The human wealth of the old and young households evolve as:

$$h_t^o = d_t^o + \frac{\gamma(1+n)}{r_t} h_{t+1}^o, \tag{26}$$

$$h_t^y = \frac{W_t}{P_t} + d_t^y + \frac{1+n}{r_t} \frac{\omega}{\Phi_{t+1}} h_{t+1}^y + \frac{1+n}{r_t} \frac{(1-\omega)\, \epsilon_{t+1}^{\frac{\rho-1}{\rho}}}{\Phi_{t+1}} h_{t+1}^o.$$

Finally, the resource constraint is given by:

$$y_t = c_t^o + c_t^y + i_t + \frac{\phi}{2} \left(\pi_t - 1 \right)^2 y_t, \tag{27}$$

These equations together with monetary policy,

$$\pi_t = \overline{\pi},$$

constitute the equilibrium condition.

C Steady state analysis

This section provides a proof of Proposition 1 by proving a series of lemmas.

Lemma 1. *The old households' MPC out of wealth in a steady state is unambiguously decreasing in γ. If $0 < \rho < 1$, mpc^o is decreasing in the real rate, r; If $\rho < 0$, mpc^o is increasing in r.*

This lemma follows obviously from equation (20).

An increase in γ acts to make the old households more patient and, at the same time, to reduce the rate of return on savings, r/γ. The latter effect on the old households'

MPC may be positive or negative, depending on the value of ρ, but the former effect always dominates the latter. In our calibration we have $\rho < 0$, and therefore the old households' MPC increases with the real interest rate.

Next we consider the relative MPC, i.e., the ratio of the old households' MPC to the young households' MPC. Let $\varepsilon := mpc^o/mpc^y$. Then from equations (20) and (21) we have

$$\frac{1}{\epsilon} = \frac{mpc^y}{mpc^o} = \frac{1 - \beta^{-\frac{1}{\rho-1}} \left(\omega + (1-\omega)\, \epsilon^{\frac{\rho-1}{\rho}}\right)^{-\frac{\rho}{\rho-1}} r^{-\frac{\rho}{\rho-1}}}{1 - \beta^{-\frac{1}{\rho-1}} \gamma r^{-\frac{\rho}{\rho-1}}}. \tag{28}$$

Lemma 2. *Suppose $\rho < 0$. Then the old households' MPC is higher than the young households', i.e., $\epsilon > 1$, in the steady state. An increase in γ results in a decrease in ϵ, regardless of the value of ρ. An increase in the real interest rate reduces ε.*

The proof is as follows. The left-hand side, $1/\varepsilon$, is decreasing in ϵ, whereas the right-hand side is increasing in it under the assumption that $\rho < 0$. Observe that the value of the right-hand side evaluated at $\varepsilon = 1$ is less than one, the value of the left-hand side at $\varepsilon = 1$. Hence, both sides are equal only if $\varepsilon > 1$. An increase in γ reduces the denominator of the right-hand side of equation (28) and makes the right-hand side bigger, reducing the value of ε that equates both sides of equation (28). Imagine that r has increased. Then $r^{-\rho/(\rho-1)}$ decreases, and both the numerator and the denominator in the right-hand side of equation (28) increase. Because $\epsilon > 1$, the numerator is smaller than the denominator, implying that the coefficient of $r^{-\rho/(\rho-1)}$ is bigger in the numerator than that in the denominator. Hence, the effect of a change in $r^{-\rho/(\rho-1)}$ is bigger for the numerator than for the denominator, and the whole right-hand side increases when r increases, resulting in lower ϵ. If instead r decreases, then ϵ is higher.

Finally, the following lemma shows the properties of mpc^y.

Lemma 3. *Given everything else equal, when the old households' MPC changes, the young households' MPC changes in the same direction but less than one-for-one with it. Given everything else equal, mpc^y is decreasing in the real rate if $0 < \rho < 1$, and mpc^y is increasing in the real interest rate if $\rho < 0$.*

The proof is as follows. We use equation (21). The left-hand side is increasing in mpc^y and the right-hand side is decreasing in mpc^y. Holding everything else equal, an increase in mpc^o raises the right-hand side for a given mpc^y, resulting an increase in mpc^y that equates the both sides. For mpc^y to rise, the ratio mpc^o/mpc^y in the right-hand side needs be larger than the value before the change. Hence, a percent change in mpc^o must be larger than a percent change in mpc^y. Because the relative MPC is bigger than one, according to the previous lemma, $mpc^o > mpc^y$, implying that a change in mpc^y must be smaller than a change in mpc^o. The response of mpc^y to the real interest rate is qualitatively the same as that of mpc^o and thus the proof is omitted.