On the Size of the Fiscal Multiplier When the Nominal Interest Rate is Zero*

R. Anton Braun

Graduate School of Economics, University of Tokyo

Yuichiro Waki

University of Minnesota

Abstract

This paper documents that the common practice of loglinearizing New Keynesian models around a perfect foresight steady state with a constant price level can induce large biases when these economies are hit with large but plausibly sized shocks. The example we consider is the size of the government purchases output multiplier when the model is shocked to induce a binding non-negativity constraint on the nominal interest rate. Using nonlinear methods we find that loglinearized solutions systematically overestimate the size of the government purchase multiplier by as much as a factor of two. After correcting these biases the government purchases multiplier for output is still well above one.

*This version: May 13, 2010. We thank Larry Christiano and Martin Eichenbaum for their comments. Any remaining errors are our own.

1 Introduction

The recent experiences of Japan and the United States with zero/near-zero nominal interest rates have raised new questions about the conduct of monetary and fiscal policy. Recent research by Christiano, Eichenbaum and Rebelo (2009) finds that fiscal policy is particularly effective when the nominal interest rate is zero in a New Keynesian model. They consider a 5% shock to the (annualized) preference discount rate that results in a binding zero lower bound on the nominal interest rate. In their setting the output multiplier associated with an increase in government purchases is about four on impact. A multiplier of this magnitude is large and their result suggests that there is a particularly important role for the fiscal authority to stabilize the economy when monetary policy is constrained by the lower bound of zero on the nominal interest rate.

Methodology-wise, this conclusion and a range of other conclusions about the dynamics of the New Keynesian model when the nominal interest rate is zero have been reached by solving a loglinearized version of the model that is centered around a zero inflation steady state. For example, Eggertsson and Woodford (2003), Christiano (2004), Braun and Waki (2006), Eggertsson (2009) and Christiano, Eichenbaum and Rebelo (2009) use loglinearized solutions to analyze the properties of New Keynesian models when the nominal interest rate is zero.

It is well known that perturbation methods are local approximations that eventually fail when shocks are sufficiently large. Research by Coenen, Orphanides and Wieland (2004) estimates a model using the U.S. data from 1980 to 1999 and finds that the probability of a shock driving the nominal interest rate to zero is very low, when the inflation target rate is set to as high as 2%. Only large shocks produce a binding zero nominal interest rate in their estimated specification. The combination of a quantitatively important shift in the equilibrium dynamics when the interest rate is zero and the fact that large shocks are required to produce this shift in the dynamics raises questions about the reliability of local perturbation methods for computing the equilibrium.

In this paper we provide such an assessment and find that loglinear methods produce large biases that overstate the size of the government purchase multiplier by as much as a factor of two.

We assess the properties of the linearized solution strategy by comparing impulse responses to exact non-linear impulse responses from a perfect foresight version of the economy that is subject to time zero shocks. The impulse responses are calculated by solving a two point boundary problem using the exact nonlinear equilibrium conditions for the perfect foresight version of the model. We assume that the economy returns to its steadystate in period $T = 300$ and use Newton's method to simultaneously solve the nonlinear equilibrium restrictions that determine prices and allocations in each period of the transition.[1] We consider two models of price adjustment: Calvo (1983) price setting model and Rotemberg (1989) costly price adjustment model. The treatment of the costs of price adjustment/dispersion has important implications for the dynamic properties of the model. Thus, when solving the Calvo specification we follow Schmitt-Grohe and Uribe (2007) and keep track of the costs of price dispersion by introducing auxiliary state variables. Parameters are chosen so that the two models of price adjustment produce the same equilibrium condition when loglinearized.

Our findings are as follows. For small shocks that increase the annualized preference discount factor by 2% , the precise form of price adjustment doesn't make a big difference and the loglinearization works well for both price adjustment specifications. For a shock of this size, the zero interest rate bound never binds under our parameterization of the model and the government purchase multiplier is small. When we increase the annualized preference discount factor by 5%, the same size as in Christiano, Eichenbaum, and Rebelo (2009), the zero interest rate bound binds, and the quality of the approximation of the loglinear solution sharply deteriorates. For example, the government purchases multiplier on output falls from about 4, when using the loglinearized solution, to about 3.3 in the Calvo specification and to about 2.5 using the Rotemberg specification. In our nonlinear setting the size of the government purchase multiplier varies with the size and the duration of the shock to government purchases. We explore the robustness of

[1] Other nonlinear solution strategies such as parameterized expectations used by Adam and Billi (2007) or the finite element method used by Wolman (2005) are difficult to apply here. The zero interest rate bound (ZIRB) is highly endogenous in our setting with capital formation. To obtain a policy function one would need to divide the state space into two regions- one where the ZIRB binds and one where it does not. That set is very difficult to characterize. For instance, we do not even know whether this set is convex. Another advantage of our approach is that it does not require function approximation which by itself introduces an approximation error.

our conclusions to these assumptions and find that, when the zero interest rate bound is binding, the loglinearization robustly yields an upward bias. The size of this bias depends on the form of price adjustment.

The upward bias associated with the loglinear solution is due to two factors. One important factor relates to the resource costs of price adjustment/dispersion. Log-linear solutions typically center the approximation around a steady state where these resource costs and their derivatives are zero. In our example, a 5% increase in the preference discount factor has a big effect on these costs. The implied resource costs of price adjustment/dispersion using the loglinear solution are as large as 15% of production. The nonlinear solution, in contrast, which explicitly recognizes these costs produces resource costs that are 3.5% of production or smaller.

In the true model, a positive shock to government purchases acts to reduce these resource costs under either form of price adjustment. This leaves more resources available for consumption and investment and the resulting government purchase multiplier for output continues to be greater than one. This effect is particularly pronounced for the Rotemberg specification. For that specification the production multiplier is often less than one.[2] It follows that the reason why the multiplier for output is greater than one is because the resource costs of price adjustment have fallen.

A second important factor is that these resource costs have a damping effect on the output response to a preference discount factor shock and this acts to attenuate the response of marginal cost. Under the loglinear solution a 5% discount factor shock lowers marginal cost by 23%. When the resource costs are recognized the size of the response of marginal cost is much smaller. A higher resource cost or price adjustment acts like a negative shock to production that attenuates the response of the markup to the preference discount factor shock.

Our research is related to work by Ascari and Rossi (2008). They also find that these two specifications of price stickiness have distinct implications when nonlinear solution methods are used. Their focus is on the dynamics after temporary and permanent shocks to the central bank's

[2] In our model output is the sum of consumption, investment and government purchases. This differs from production because output is net of the resource costs of price adjustment.

target inflation rate, whereas ours is on preference shocks that generate large responses when the nominal interest rate hits its lower bound of zero.

Our research is also related to Braun and Körber (2010). They consider an application of the New Keynesian model to Japanese data. Their work suggests that the size of the preference discount factor shocks considered here are empirically relevant for Japan in the sense that shocks of this size are required to induce a binding zero nominal interest rate. They also find that recognizing the resource costs of price adjustment has important implications for the response of output to labor tax shocks and technology shocks.

The remainder of the paper proceeds as follows. In Section 2 we describe the model. In Section 3 we describe how we solve the model. Section 4 describes the computational experiments we perform. Section 5 reports the results and Section 6 contains our concluding remarks.

2 The Model

We consider a perfect foresight version of the New Keynesian model with capital accumulation of Christiano, Eichenbaum and Rebelo (2009) (see also Christiano (2004), and Braun and Waki (2006) for similar models.).

Household's problem

The representative household values alternative sequences of final good consumption c_t, leisure $1-h_t$, and end-of-period real money balance M_{t+1}/P_t using the present value utility function:

$$\sum_{t=0}^{\infty} \beta^t \big(\prod_{s=0}^{t} d_s \big) \big\{ u(c_t, 1-h_t) + \Upsilon\big(\frac{M_{t+1}}{P_t}\big) \big\}. \qquad (1)$$

The period utility function is non-separable in consumption and leisure: $u(c, 1-h) = \frac{(c^{\nu}(1-h)^{1-\nu})^{1-\sigma}-1}{1-\sigma}$. Non-separable preferences help generate large government purchase multipliers, with parameter values that make the marginal utility of consumption increase with labor. Function Υ is nondecreasing, concave, and has a satiation point of real money balance, i.e. there exists $\overline{m} < \infty$ such that $\Upsilon'(m) = 0$ for all $m \geq \overline{m}$.

The one-period discount factor varies over time. The preference discount factor between period t and $t+1$ is βd_{t+1} where d_{t+1} is an exogenous shifter. We normalize d's so that its steady state value is one. In our experiments d_t's are higher than one by a given constant only for initial finite periods.

The budget constraint for the household for all $t \geq 0$ is:

$$c_t + x_t + \frac{M_{t+1} + B_{t+1}}{P_t} = \frac{M_t + (1+R_t)B_t}{P_t} + \int_0^1 \frac{\Xi_t(i)}{P_t} di + T_t + r_t k_t + w_t h_t, \tag{2}$$

where x is investment, B is the riskless bond holding, $\Xi(i)$ is a nominal profit receipt from intermediate firm i, T is a lump sum transfer, P_t is the nominal price of final good, R_t is a net nominal interest rate of the government bond from period $t-1$ to t, and r_t and w_t denote the rental rates of capital and labor, respectively. The household also faces a no-Ponzi game condition.

The capital accumulation equation: for all $t \geq 0$ is:

$$k_{t+1} = (1-\delta)k_t + x_t - \Phi(\frac{x_t}{k_t})k_t. \tag{3}$$

Φ is the capital adjustment cost function, which we assume is a quadratic function $\Phi(x/k) = \frac{\sigma_I}{2}(x/k - \delta)^2$ in the numerical exercises that follow. Then the household's problem is to maximize (1) by choice of the sequence $\{c_t, h_t, k_t, M_{t+1}, B_{t+1}, x_t\}$ subject to (2), (3) and $k_0 = \overline{k}_0, M_0 = \overline{M}_0, B_0 = \overline{B}_0$, taking prices and transfers as given.

Final good producer

Perfectly competitive final good producers use intermediate goods to produce a single final good with the production function:

$$y_t = \Big[\int_0^1 y_t(i)^{\frac{\theta}{\theta-1}}\Big]^{\frac{\theta-1}{\theta}}.$$

Optimality and zero profits imply that the price of the final good is of the form

$$P_t = \Big[\int_0^1 p_t(i)^{1-\theta}\Big]^{\frac{1}{1-\theta}},$$

where $p_t(i)$ is the nominal price of intermediate good i in period t. Also for a given production level y_t, the demand function for the intermediate good i is given by $y_t(i) = \big(p_t(i)/P_t\big)^{-\theta} y_t$.

Intermediate goods firms

We consider two forms of costly price adjustment: Rotemberg-type quadratic price adjustment costs and the Calvo-type price setting. These two specifications are observationally equivalent when the model is linearized. We will see that there are some important differences though between the two specifications when one uses an exact solution method to solve the model instead.

Calvo specification of costly price adjustment

If firm i can change prices in t, it maximizes

$$\sum_{j=0}^{\infty} \Lambda_{t,t+j} \frac{1}{P_{t+j}} \gamma^j \Big\{ p_t(i) y_{t+j}(i) - (1-\tau_\chi) P_{t+j} mc_{t+j} y_{t+j}(i) \Big\} \tag{4}$$

subject to the demand function $y_{t+j}(i) = \big(\frac{p_t(i)}{P_{t+j}}\big)^{-\theta} y_{t+j}$, where $\Lambda_{t,t+j}$ is the discount factor between periods t and $t+j$ that in equilibrium equals to the representative household's: $\beta^j \big(\prod_{s=1}^{j} d_{t+s}\big) \frac{u_{c,t+j}}{u_{c,t}}$. Parameter γ denotes the probability that a firm can not change its price. The variable mc is the marginal cost of producing one unit of intermediate good and satisfies $mc_t = \frac{r_t^\alpha w_t^{1-\alpha}}{\alpha^\alpha (a-\alpha)^{1-\alpha} A_t^{1-\alpha}}$. τ_χ is a tax that corrects the monopolistic rent. Its value is set so that the steady state profits for intermediate good producers are zero. This implies $1-\tau_\chi = (\theta-1)/\theta$.

We solve the nonlinear equilibrium condition using a device developed by Schmitt-Grohe and Uribe (2006) that introduces auxiliary state and jump variables. Details can be found in the appendix.

Rotemberg specification of costly price adjustment

Firm i maximizes

$$\sum_{t=0}^{\infty} \Lambda_{0,t} \frac{1}{P_t} \Big\{ p_t(i) y_t(i) - (1-\tau_\chi) P_t mc_t y_t(i) - P_t \Gamma\big(\frac{p_t(i)/p_{t-1}(i)}{1+\pi_{ss}}\big) y_t^g \Big\} \tag{5}$$

subject to the demand function $y_t(i) = \big(\frac{p_t(i)}{P_t}\big)^{-\theta} y_t$. The term y_t^g in the above expression denotes the aggregate production (gross output) and is given by $y_t^g = k_t^\alpha (A_t h_t)^{1-\alpha}$. This specification is taken from Braun and Waki (2006). The adjustment cost function Γ has a quadratic form: $\Gamma(z) = \frac{\gamma^{AC}}{2}(z-1)^2$.

Although this formulation allows for the possibility that the price adjustment costs are centered at a nonzero inflation rate, we set $\pi_{ss} = 0$ and $1 - \tau_\chi = \frac{\theta-1}{\theta}$ in our experiments. Under this assumption a stable price level is desirable whenever possible. We choose γ_{AC} and γ so that the loglinear equilibrium conditions are identical for the two specifications.

Resource Cost of Price Stickiness

In the Appendix we show that the resource constraint in the Calvo model in a symmetric equilibrium is given by

$$y_t = c_t + x_t + g_t = \frac{1}{PD_t^{-\theta}} k_t^\alpha (A_t h_t)^{1-\alpha},$$

where $PD_t = [\int_0^1 \big(p_t(i)/P_t\big)^{-\theta} di]^{-1/\theta}$. In a symmetric equilibrium the resource constraint in the Rotemberg specification is

$$y_t = c_t + x_t + g_t = \Big\{1 - \Gamma(\frac{1+\pi_t}{1+\pi_{ss}})\Big\} k_t^\alpha (A_t h_t)^{1-\alpha}.$$

Government

The fiscal policy in this economy is Ricardian and the same as in Braun and Waki (2006) and Christiano, Eichenbaum and Rebelo (2009). Lump-sum taxes adjust to balance the government budget constraint when government purchases are increased.

Monetary Policy

The monetary policy follows the Taylor rule:

$$R_{t+1} = \max\left\{0, (1+R_{ss})(\frac{1+\pi_t}{1+\pi_{ss}})^{\rho_\pi} - 1\right\}.$$

Equilibrium Conditions

Our definition of a competitive equilibrium is the same as in Braun and Waki (2006). It is standard, except that we focus on a particular class of equilibria in which the zero bound is hit at most once and for a finite and consecutive number of periods. We also choose the equilibrium in which the nominal interest rate is zero for the shortest number of periods. These requirements rule out multiple equilibria.

The most important characteristic of the equilibrium is that it includes a specification of the period when the nominal interest rate first falls to zero, S, and a second period after which the nominal interest rate permanently rises above zero, T; $R_{t+1} = 0$ if and only if $t \in \{S, ..., T\}$. The interested reader is referred to Braun and Waki (2006) for more details.

The equilibrium conditions are reported in the Appendix.

3 Solution methods

Our basic strategy for computing the equilibrium is a standard one for solving nonlinear perfect foresight models: We solve a two point boundary problem, and guess-and-verify (S, T). We assume that the economy reaches the steady state within a finite number of periods, make a guess on (S, T), and solve the nonlinear equilibrium condition by Newton's method for an equilibrium sequence of quantities and prices. We then check whether the quantities and prices computed in this way in fact constitute an equilibrium. We solve the loglinearized version of the model in a similar way but exploit the linearity of the system to economize on the computational burden.[3]

Nonlinear Solution

The specific system of equations used when solving the two specifications of costly price adjustment is reported in the technical appendix. The terminal period is set to 300. It typically takes about 4-10 seconds to compute the equilibrium for the Rotemberg specification. The Calvo specification can take as much as one minute to solve for larger sized shocks. The reason for this is that we use a fixed step size in each Newton update and it is necessary to set it low in order to insure convergence.

Loglinear solution method

Braun and Waki (2006) exploit univariate linear forecasting formulas described in Hansen and Sargent (1980). Here we solve a large, sparse system of linear equations. An advantage of this approach is that it uses exactly the same equilibrium conditions as the nonlinear method. This makes it possible to attribute any differences in the two solutions to the loglinear approximation.

As a check we have compared the impulse responses obtained by solving the sparse-system to impulse responses reported in Braun and Waki (2006) and found that the two methods yield virtually indistinguishable solutions.

4 Computational Experiments

Discount factor shock

We now describe the specific experiments we perform. We start by subjecting the economy to a persistent increase in the preference discount factor d_{t+1}. This shock lowers the real interest rate and thereby pushes the nominal interest rate down to zero. We assume that the discount factor shock $d_{t+1} - 1$ takes a positive, constant value for a known number of periods from $t = 0$, and

[3]For the linear system we use Gaussian elimination instead of Newton's algorithm.

becomes zero afterward. In the baseline experiment the discount factor shock returns to zero in $t = 10$.

Government purchases

We first solve for the equilibrium with government purchases set to their steady state value: $g_t = g_{ss}$ for all t and compute the equilibrium allocations. Then we increase government purchases and report how the equilibrium allocation changes.

We assume that g_t increases for an exogenously fixed and finite number of periods. We don't use an AR(1)-type of law of motion for government purchases. An AR(1) law of motion generally yields smaller government purchase multipliers as compared to the results reported here.[4]

One difference compared to Christiano, Eichenbaum and Rebelo (2009) is that we don't allow g_t to depend the value of the nominal interest rate. Christiano, Eichenbaum and Rebelo (2009) assume that g_t increases by a fixed amount "for as long as the zero bound binds." This assumption creates some problems for our experiments. We will show later on that the number of periods that the nominal rate is zero differs across the various solution methods and different models of price adjustment. If we use the method of CER the resulting sequences of government purchases will also differ across specifications. This makes it more difficult to conduct comparisons across the various specifications.[5]

Two Multipliers

For both specifications of price stickiness measured output or GDP is:

$$y_t := c_t + x_t + g_t = (1 - \kappa_t) k_t^{\alpha} (A_t h_t)^{1-\alpha},$$

[4]See Braun and Körber (2010) for an analysis of the case with A.R. 1 shocks

[5]Another, technical reason for our assumption has to do with the fact that we consider large shocks to government purchases. Sufficiently large shocks to g can push the economy out of the zero interest rate bound. However, this is inconsistent with the conjectured equilibrium which posits that g is only high during periods when the nominal interest rate is zero. In other words, for sufficiently large shocks to g there is no equilibrium of the form they conjecture.

where $\kappa_t = \Gamma(\frac{1+\pi_t}{1+\pi_{ss}})$ in the Rotemberg model and $\kappa_t = 1 - \frac{1}{PD_t^{-\theta}}$ in Calvo specification. We call $\kappa_t k_t^\alpha (A_t h_t)^{1-\alpha}$ the total resource costs of price dispersion/adjustment costs.

We report multipliers for output, y_t, and production (gross output), $y_t^g = k_t^\alpha (A_t h_t)^{1-\alpha}$. When loglinearized, κ disappears from the resource constraint and these two objects behave in the same way up to numerical errors. In the nonlinear model the behavior of κ plays an important role in the determination of the size of the government purchase output multiplier.

Model Parameterization

To facilitate comparison, we report results using the same model parameterization as Christiano, Eichenbaum and Rebelo (2009). One exception is the capital share parameter α, which we set to 0.4, because their choice of this parameter is not reported. We report parameter values in Table 1.

5 Results

In this section we report impulse responses to government purchase shocks for the two models of cost adjustment. We summarize the results in two ways. We use tables to document the impact response of output y_t, production y_t^g, and other key variables to shocks of government purchases of alternative sizes and durations. We also use figures to report a more comprehensive set of impulse response functions.

Tables 2 through 5 report impact responses. Tables 2 and 4 contain results for the Calvo specification, and Tables 3 and 5 contain results for the Rotemberg specification. We consider 1% and 10% increases in government purchases. The 1% shock is chosen to control for the duration of the episode of zero nominal interest rates. A shock of this size to government purchases does not alter the number of periods that the nominal interest rate is zero for any of the specifications considered here.

We also report results for a 10% shock to government purchases. Shocks of this size typically reduce the number of periods that the nominal interest rate is zero. Each table reports results for a shock to government purchases of a fixed size and varies the size of the discount factor shock. For purposes of comparison we report discount factor shocks of; 2%, 5%, and 6%. A 5% discount factor shock corresponds to a baseline reported in CER (2009).[6]

In addition to the two impact multipliers for production and output, we also report the period when the nominal interest rate first binds, S, the period where the constraint last binds, T, the resource costs of price dispersion, κ, and the real unit costs of intermediate good production, mc, for scenarios with and without the government purchases shock. For the loglinearized solution the resource costs, κ, are "implied" values that are calculated from prices and inflation rates, and for the nonlinear solution these are the actual resource costs that enter the resource constraint. We report mc because $(1 - mc)$ acts as a wedge between the marginal rate of transformation and marginal rate of substitution and is thus a good measure of one of the distortions to the economy.

5.1 A 1% shock to government purchases

Calvo specification

Table 2 reports the results for a 1% shock to government purchases for the specification with Calvo price setting.

We compute the multipliers in the loglinearized economy by first transforming log-deviations into levels, and then calculating multipliers using the levels. This procedure induces some small differences in the multiplier for output and production in the loglinearized solution.

For moderate values of the discount factor shock that are of a magnitude of 3% or less the loglinear solution works reasonably well in the sense that the government purchase multiplier is of about the same magnitude under the loglinear and non-linear solution. It is not surprising that the multiplier is low for a 2% shock, because the zero interest rate bound (ZIRB) never binds for

[6] For discount factor shocks that are less than 4% the monetary authority could stabilize the price level completely by setting R_{t+1} so that $d_{t+1}(1 + R_{t+1}) = 1 + R_{ss}$ and thereby correcting the dynamic distortion created by costly price adjustment. In other words, for shocks that are less than 4% the zero interest rate bound would not restrict the actions of a policy maker seeking to stabilize the price level. In this sense the zero interest rate bound only limits the actions of the monetary authority for shocks that are larger than 4%.

discount factor shocks of this size. However, the value reported here is still much lower than the government purchase multiplier reported in Christiano, Eichenbaum and Rebelo (2009). They report a value of about 1. We will see below, when we report results for the figures, that this distinction is due to the fact that the dynamic response of output is hump-shaped in this scenario.

Increasing the size of the discount factor shock from 2% to 3% only has a moderate effect on discounting; βd increases by about 0.0025. This small change in discounting has a very big effect on the size of the output multiplier which increases from around 0.1 to 1.8.

For the case of a 5% discount factor shock, the output multiplier is around 4 for the loglinear solution. This value is consistent with results reported in Christiano, Eichenbaum and Rebelo (2009) (See their Figure 6). Whereas the multiplier obtained using our nonlinear solution method is 3.3. The size of the bias using the loglinearized solution is about 0.66. The approximation error for the loglinear is even larger for production. For this variable the loglinear multiplier is 3.9 whereas the nonlinear multiplier is 2.2.

When the shock size is increased to 6%, the multiplier increases further to more than 5 under the loglinearization, while it decreases to 3.1 in the nonlinear solution.

Note also that the implied value of κ for the loglinear solution increases rapidly with the size of the discount factor shock. If one derives output from production using the implied resource costs, $(1-\kappa)F(k, Ah)$, the implied value of the multiplier is huge! For instance, if we use the scenario of a 5% shock to the preference discount factor the implied government purchase output multiplier is 12.2. From this it is clear that this approximation error is a serious issue that does not have a simple remedy. A simple fix like plugging the implied resource costs from the loglinear solution into the true resource constraint produces an even larger bias.

Not surprisingly, when these resource costs are explicitly modeled agents take actions to reduce them. This results in smaller values of κ in the nonlinear solution. For a discount factor shock of 5% the explicit resource costs reported in Table 2 fall from 16% of production to 2% of production when they are explicitly modeled. This yields lower impact multipliers for both output and production.

One way to measure the dynamic distortion to the economy induced by costly price adjustment is to check how far mc is from 1. According to Table 2 the size of this distortion is large for the loglinear solution when the ZIRB binds. A 5 % discount factor shock reduces marginal cost from 1 to 0.77. The marginal cost distortion is smaller in the nonlinear solution with a value of 0.82.

Rotemberg specification

Next we turn to the Rotemberg specification. The loglinear solution to this model is equivalent to the Calvo specification. However, the nonlinear solutions are quite different. In particular, for the shock sizes considered here the multipliers in the Rotemberg model are generally smaller than those in the Calvo specification.

Table 3 reports results for a 1% shock to government purchases using the Rotemberg specification. Observe that the approximation errors associated with the loglinear solution are even larger here. The government purchase output multiplier using the exact solution is now 2.5 as compared to about 4 for the loglinear solution for a 5% discount factor shock. The bias from the loglinearization is now around 1.5. Interestingly, approximation is actually particularly poor for smaller shocks. For example, for a 3% shock, the true output multiplier is less than one and less than half the size of what the loglinear solution suggests. For the Calvo specification, in contrast, the two solutions are very close in this scenario.

Although not reported in Table 3 due to space considerations we have also performed experiments with larger discount factor shocks. The bias can also be quite larger. For instance, in the case of a discount factor shock of 8% the output multiplier falls by more than half when using the nonlinear solution.

Note also that the production multipliers are consistently less than 1 in Table 3 for the nonlinear solution. For this specification of price adjustment the fact that the multiplier on output is greater than one comes from lower resource costs of price adjustment (lower κ). To see why this is the case note that the output multiplier can be expressed as:

$$\frac{dy}{dg} \approx (1-\kappa)\frac{dy^g}{dg} - y^g\frac{d\kappa}{dg}$$

The steady state value of y^g is around 1.8, which is much larger than $1-\kappa$. It follows that the only way for the output multiplier to be large is for $\frac{d\kappa}{dg}$ to fall by a lot.

A comparison of the loglinear solution results for this specification with the results reported in Table 2 shows that the impact responses are nearly identical. This is by design; The adjustment cost parameter has been chosen to produce equivalent loglinear solutions.

There are, however, some important differences between the nonlinear results reported in Tables 2 and 3. For preference shocks of 5% the government purchase output multipliers using Rotemberg cost adjustment are lower than those for the Calvo model (2.5 as compared to 3.3). Note also that the resource costs of price adjustment are larger than the resources costs of price dispersion. However, the distortion in marginal cost $(1-mc)$ is higher under Calvo price adjustment (mc is 0.82 in the Calvo specification and 0.89 in the Rotemberg specification).

Intuition for a model without capital

We have shown that loglinear solutions can exhibit large biases and have argued that a principal reason for these biases is that the loglinear solution abstracts from the resource costs of price dispersion/adjustment. In this section we provide some intuition about why and how variable resource costs κ matter. In particular, we will illustrate that recognizing variation in κ acts to ameliorate the negative effects of a binding ZIRB using graphical methods. The graphs that we report would emerge from, for instance, a model with Rotemberg price adjustment costs with no capital accumulation and an i.i.d. transition probability for the preference discount factor shock along the lines of the model considered by e.g. Eggertsson and Woodford (2003).

Figure 1 depicts a subset of equilibrium condition in the steady state on (c,h)-plane. We draw the resource constraint with $\kappa = 0$ and $g = g_{ss}$, $MRS_{c,h} = w = mcf'(h)$ with $mc = 1$, and the iso-marginal utility of consumption curve $u_c(c, h) = u_c(c_{ss}, h_{ss})$. In a steady state equilibrium these curves intersect with each other at point A.

Suppose now that the economy is hit by a discount factor shock and the nominal interest rate can't absorb it due to the ZIRB. This leads to a higher marginal utility of consumption. Figure 2 depicts what happens when both κ and g are unchanged. The iso marginal utility curve shifts downward (arrow (1)). The resource constraint stays the same, and the new equilibrium values of (c, h), occur at point B in the figure. As a result, marginal cost, mc, has to decrease so that the other curve also passes through point B (the arrow (2)). Intuitively, aggregate demand decreases because of the discount factor shock and thus the total supply of goods must also fall to in order for markets to clear. This works via a decrease in mc. A decrease in marginal costs lowers wages and lowers work effort of households.

Figure 3 depicts what happens when either κ or g is increased. (The arrows labeled (3) illustrate the dynamics.) Both shocks work in a similar way- a fraction of what is produced in the economy gets thrown out. This reduces the size of the change in the markup needed to get households to supply less labor. The new equilibrium occurs at point C. This point has higher consumption, c, and higher labor input, h, as compared to point B.[7] This is how shocks to g and/or κ ameliorate the response of marginal cost when the preference discount factor is increased.

Note also that a given percentage change in κ can have bigger implications than an equivalent percentage change in g. To see this, let us compare two situations: (a) κ increases by 1% from its steady state value of zero; (b) government purchases increases by 1% from its steady state value. In the first scenario, $(1 - \kappa)f(h) - g$ decreases by approximately $f(h_{ss})$, whereas in the second it decreases approximately by only $g_{ss} < f(h_{ss})$.

It is clear from this graphical analysis that abstracting from movements in κ can, in principal, affect the output response.

What happens when investment can vary? Positive discount factor shocks make the household

[7]Whether c increases or decreases depends on the slope of the iso-marginal utility curve. We draw it as an upward sloping curve because it is so under our parameterization. If, for example, the period utility is separable in consumption this curve is flat and c doesn't increase.

more patient, which induces substitution between consumption and investment if the rental rate of capital in the near future doesn't fall. When the zero interest rate bound binds for a long time, an initial drop in mc (and hence in r) is large enough to discourage the households from this substitution, at least initially. As time passes, mc increases toward one and the households start accumulating capital.

5.2 A 10% shock to government purchases

Tables 4 and 5 report impact responses to a 10% shock to government purchases.

Table 4 reports the results using the specification with Calvo price setting. Notice that the government purchase output multipliers computed using our nonlinear method are lower than those in Table 2. For a 5% discount factor shock the size of the output multiplier declines from about 4 to 3.1 in the loglinearized solution and from 3.3 to 2.3 using the nonlinear solution. The size of the bias is still around 0.66. The production multiplier is also smaller but still larger than one.The reason for this reduction in the multipliers is that the duration of the ZIRB has fallen from 5 quarters to 3 quarters. The larger shock to government purchases induces a larger positive response in prices and this in turn lowers the distortion in marginal cost from 0.83 in Table 2 to 0.89 in Table 3 for the nonlinear solution.

Table 5 reports results for the Rotemberg specification. A larger shock to government purchases reduces the "true" output multiplier in most cases. For instance for a 5% discount factor shock the government purchase output multiplier for the nonlinear solution declines from 2.5 to 1.8. A value of this magnitude lies in the range of estimates reported by e.g. Romer and Bernstein (2009) who estimate the government purchase multiplier to be about 1.6.

For the Rotemberg specification the size of the production multiplier in the nonlinear solution continues to be well less than one and to fall with the size of the shock. As noted above, for this specification, lower resource costs of price adjustment are playing a key role in producing a government purchase output multiplier that is greater than one.

5.3 Dynamics of the economy with and without government purchases shock

We next use graphs to document the dynamic response of a broader range of variables to shocks in the discount factor and government purchases.

Figures 4 through 6 report impulse responses using the loglinear and nonlinear solution methods. Each figure consists of two panels; The left panel reports results for the loglinear solution; The right panel reports using the nonlinear solution. Each panel contains solid and dashed lines. Solid lines report responses to a discount factor shock with the government purchases shock set to zero. The dashed lines show how the impulse responses functions change when government purchases are increased by 10% at the same time.

Figure 4 reports the impulse responses for the Calvo specification with a 5 % discount factor shock. Although the qualitative shapes of the responses are quite similar for both solution methods, the loglinearization tends to overstate the size of the responses. For example, without government purchases shocks, hours respond around 15 % using the loglinear solution, whereas the "true" response is closer to 8% . The one exception to this general pattern is the inflation rate. Inflation exhibits a larger response using the nonlinear solution. A comparison of the dashed lines in the two panels reveals a similar picture. The shapes are very similar but the magnitude of the responses is generally more damped when using the nonlinear solution method.

Figure 5 reports the response of the same variables to a 2% shock in the discount factor. For a shock of this size the nominal interest is positive in all periods. In this situation the loglinear solution works well. The responses under the two solution methods are very similar in both shape and magnitudes.

A comparison of Figure 4 and 5 indicates that the zero bound has some very important effects on the dynamics of the model and that these differences are robust to the solution method. Investment, for instance, increases when hit by a discount factor shock in Figure 5, while it decreases on impact in Figure 4. The shape of the responses of output, production and hours is also quite different in the two Figures. In Figure 5 these variables all peak in period 9. Whereas in Figure 4 they peak on impact. The principal reason for these differences is the response of marginal costs. The response of mc is much smaller when the ZIRB is not binding. $(1-mc)$ acts as a wedge between the marginal rate of substitution and the marginal rate of transformation, and can be interpreted as a "tax" on factor prices. When $(1-mc)$ is big, households are discouraged from accumulating capital and supplying labor. $(1-mc)$ responds by more than 15% on impact in Figure 4 and only by around 2% in Figure 5. This explains the qualitative difference in the responses.

Figure 6 reports impulse response functions for a five percent shock to the preference discount factor for the Rotemberg specification. One important difference between the right and left panels concerns the response of hours. The difference between the loglinear solution and the nonlinear solution is quite pronounced. Hours fall by nearly 15% when using loglinear methods to solve the model and by only about 2% when using our nonlinear method. This difference in the hours response translates into some large differences for production and output. The investment response is also much smaller for the nonlinear solution. These differences in investment and hours can be traced back to the response of marginal cost. It falls by over 20% in the case of the loglinear solution as compared to about 10% in case of the nonlinear method.

For a 2 percent shock to the preference discount factor the loglinear solution works well and the impulse responses for both the loglinear and non-linear solution are very close for all variables. For considerations of space we don't report the impulse responses for this case.

It is also interesting to compare the nonlinear solution under the two forms of price adjustment. Compare the right panel in Figure 6 with the right panel in Figure 4. The size of the responses are generally smaller in the quadratic Rotemberg specification. The response of mc, for instance, is about 15% in Figure 4 as compared to 10% in Figure 6 and the inflation response is about half as large.

5.4 Shorter duration for government purchases shock

Our results assume that government purchases is high for 10 periods which is much longer than the number of periods where the ZIRB binds. We now examine how the results change if we assume that government purchases are high for only 5 periods instead. Table 6 and 7, report results for this scenario. The government purchases shock is 1% and in Table 6 and 10% in Table 7.

For the 1% government purchases shock scenario reducing the duration of the shock reduces the size of the multiplier for both specifications of price adjustment costs.

When the size of the government purchases shock is 10 % instead, the results are somewhat surprising. When using loglinear methods to solve the model, the size of the government purchase output multiplier is also now smaller. However, this is not true for the nonlinear model. For both the Calvo and Rotemberg specifications the multipliers reported in Table 7 are higher than the corresponding multipliers in Tables 4 and 5.

5.5 Summary

Taken together these results indicate that there are good reasons to be concerned about the quality of loglinear approximations when considering shocks that suddenly drive the nominal interest rate to zero. When the zero interest rate bound is binding, the loglinear approximations tend to exaggerate the government purchases multiplier for both specification of price stickiness. Moreover, assumptions that are innocuous when using loglinear methods such as e.g. the form of price adjustment costs are actually quite important when considering shocks that drive the nominal interest rate to zero. The two forms of price adjustment have quite different quantitative implications. It should be emphasized though that the government purchase output multiplier is still well above one for either form of price adjustment. Even here though there is an important difference between the two specifications of price adjustment. Under the Calvo specification the response of production is also greater than one. However, when using the Rotemberg specification magnified response of output is due to lower resource costs of price adjustment.

6 Concluding Remarks

In this paper we have shown that the size of the government purchase multiplier depends both on the form of price adjustment costs and also the way the model is solved. The common practice of loglinearizing the model around a perfect foresight steady state works well for small shocks. But for large empirically relevant shocks it breaks down. Our results suggest that the problem is due to the fact that the resource costs of price adjustment disappear from the resource constraint when the model is linearized around a steady state with price stability. We have documented that in the presence of large shocks the biases associated with the linear solution can be quite large. We have focused on the case of the ZIRB. However, the biases we have documented here are not confined to this situation. Large biases can also arise in other situations too. For instance, the biases can also be large when simulating a New Keynesian model through an episode of moderate inflation about 5%, if one assumes that the steady state inflation rate is zero as is commonly done in the literature.

References

[1] Adam, Klaus and Billi, Roberto M. (2007) "Discretionary monetary policy and the zero lower bound on nominal interest rates," *Journal of Monetary Economics* vol. 54(3), pages 728-752.

[2] Ascari, Guido and Lorenza Rossi. (2008) "Long-run Phillips Curve and Disinflation Dynamics: Calvo vs. Rotemberg Price Setting." mimeo.

[3] Benhabib, J., Stefanie Schmitt-Grohe and Martin Uribe (2001) "The Perils of Taylor Rules." *Journal of Economic Theory* Vol. 96, 40-69.

[4] Braun, R. Anton, and Lena Mareen Körber (2010), "New Keynesian Dynamics in a Low Interest Rate Environment" mimeo.

[5] Braun, R. Anton, and Yuichiro Waki (2006), "Monetary Policy during Japan's Lost Decade" *The Japanese Economic Review* Vol. 57, No. 2, 324-344.

[6] Calvo, Guillermo A. (1983). "Staggered prices in a utility-maximizing framework." *Journal of Monetary Economics* 12(3), pp.383-98.

[7] Christiano, Lawrence (2004), "The Zero-Bound, Low Inflation, and Output Collapse," mimeo.

[8] Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2009), "When is the Government Spending Multiplier Large?" NBER Working Paper No. 15394.

[9] Coenen, G., A. Orphanides and V. Wieland, "Price Stability and Monetary Policy Effectiveness when Nominal Interest Rates are Bounded at Zero", Advances in Macroeconomics, 2004, 4(1), 1187-1187.

[10] Eggertsson, Gauti, and Michael Woodford (2003) "The Zero Interest Rate Bound and Optimal Monetary Policy." Brookings Panel on Economic Activity.

[11] Eggertsson, Gauti B. (2009), "What Fiscal Policy is Effective at Zero Interest Rates?" Federal Reserve Bank of New York Staff Report, No. 402.

[12] Romer, Christina and Jared Bernstein (2009), "The Job Impact of the American Recovery and Reinvestment Plan." January 8, 2009.

[13] Rotemberg, Julio (1996), "Prices, Output and Hours: An Empirical Analysis based on a Sticky Price Model", *Journal of Monetary Economics*, 37, pp. 505-33.

[14] Schmitt-Grohe, Stephanie, and Martin Uribe (2007), "Optimal Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics* 54, 1702-1725.

[15] Wolman, Alexander (2005) "Real Implications of the Zero Bound on Nominal Interest Rates." *Journal of Money, Credit and Banking* 37, no. 2, 273-296.

7 Tables and Figures

Table 1: Parameter Values

The capital share in the production function	$\alpha = 0.4.$
The preference discount factor	$\beta = 0.99$
The depreciation rate on capital	$\delta = 0.02.$
The elasticity of substitution between intermediate goods	$\theta = 1 + 1/0.15.$
The consumption weight in preferences	$\nu = 0.29.$
The relative risk aversion coefficient	$\sigma = 2.$
The adjustment cost parameter for capital	$\phi_k = 17.$
The subsidy to intermediate goods producers	$\tau_{mc} = 1 - \frac{\theta-1}{\theta}.$
The unconditional probability of not being able to change prices	$\gamma = 0.85.$
The growth rate of TFP	$\mu = 0.0.$
The Taylor rule elasticity on inflation	$\rho_\pi = 1.5.$
The steady-state nominal interest rate	$R_{ss} = \frac{1}{\beta} - 1.$
The steady-state government purchases share of output	$g_{ss}/y_{ss} = 0.2.$
The steady-state level of TFP.	$A = 1.0.$

Table 2: Comparison across solution methods: Calvo specification, 1% g shock

	d shock (APR)	g multiplier on impact output y	production y^g	ZIRB (S,T)	resource costs κ on impact	Real unit costs mc on impact
loglinear	2%	0.0790	0.0799	(-,-)→(-,-)	0.0068→0.0060	0.9777→0.9762
	3%	1.8428	1.8148	(0,2)→(0,2)	0.0211→0.0180	0.9359→0.9422
	4%	3.1767	3.1226	(0,4)→(0,3)	0.0641→0.0546	0.8582→0.8698
	5%	3.9650	3.9103	(0,4)→(0,4)	0.1558→0.1365	0.7669→0.7809
	6%	5.2409	5.2315	(0,5)→(0,5)	0.3058→0.2709	0.6721→0.6898
nonlinear	2%	0.0431	0.0001	(-,-)→(-,-)	0.0008→0.0007	0.9819→0.9801
	3%	1.7690	1.5619	(0,2)→(0,2)	0.0028→0.0024	0.9407→0.9463
	4%	2.5998	2.0412	(0,3)→(0,3)	0.0086→0.0074	0.8792→0.8868
	5%	3.3062	2.1665	(0,4)→(0,4)	0.0192→0.0168	0.8239→0.8324
	6%	3.1063	1.5631	(0,4)→(0,4)	0.0350→0.0316	0.7793→0.7856

-The 5% d shock corresponds to the experiment considered in Christiano, Eichenbaum and Rebelo (2009).
- The κ's for the loglinearized solutions are implied values.
-The symbol "→" indicates a change in the value of the variable moving from an equilibrium without a g shock and to one with g shock.

Table 3: Comparison across solution methods: Rotemberg specification, 1% g shock

	d shock (APR)	g multiplier on impact output y	production y^g	ZIRB (S,T)	resource costs κ on impact	Real unit costs mc on impact
loglinear	2%	0.0785	0.0794	(-,-)→(-,-)	0.0046→0.0041	0.9777→0.9762
	3%	1.8420	1.8139	(0,2)→(0,2)	0.0137→0.0117	0.9359→0.9422
	4%	3.1879	3.1338	(0,4)→(0,3)	0.0392→0.0337	0.8581→0.8697
	5%	3.9656	3.9110	(0,4)→(0,4)	0.0913→0.0804	0.7668→0.7808
	6%	5.2428	5.2335	(0,5)→(0,5)	0.1808→0.1590	0.6719→0.6896
nonlinear	2%	0.1642	-0.0223	(-,-)→(-,-)	0.0037→0.0034	0.9826→0.9809
	3%	0.8369	0.4825	(0,2)→(0,1)	0.0084→0.0077	0.9668→0.9677
	4%	1.9113	0.9962	(0,3)→(0,3)	0.0188→0.0169	0.9300→0.9338
	5%	2.4741	0.9920	(0,4)→(0,4)	0.0350→0.0320	0.8934→0.8978
	6%	3.0629	0.9105	(0,5)→(0,4)	0.0553→0.0508	0.8620→0.8667

-The 5% d shock corresponds to the experiment considered in Christiano, Eichenbaum and Rebelo (2009).
- The κ's for the loglinearized solutions are implied values.
-The symbol "→" indicates a change in the value of the variable moving from an equilibrium without a g shock and to one with g shock.

Table 4: Comparison across solution methods: Calvo specification, 10% g shock

	d shock (APR)	g multiplier on impact output y	production y^g	ZIRB (S,T)	resource costs κ on impact	Real unit costs mc on impact
loglinear	2%	0.1274	0.0766	(-,-)→(-,-)	0.0068→0.0013	0.9777→0.9634
	3%	0.8138	0.7580	(0,2)→(-,-)	0.0211→0.0060	0.9359→0.9526
	4%	2.0365	1.9648	(0,4)→(0,1)	0.0641→0.0163	0.8582→0.9264
	5%	3.0836	3.0032	(0,4)→(0,2)	0.1558→0.1365	0.7669→0.8741
	6%	3.9178	3.8492	(0,5)→(0,3)	0.3058→0.0946	0.6721→0.8050
nonlinear	2%	-0.0075	-0.0427	(-,-)→(-,-)	0.0008→0.0001	0.9819→0.9619
	3%	0.7820	0.6778	(0,2)→(-,-)	0.0028→0.0007	0.9407→0.9551
	4%	1.8336	1.5204	(0,3)→(0,1)	0.0086→0.0021	0.8792→0.9322
	5%	2.4260	1.7770	(0,4)→(0,2)	0.0192→0.0053	0.8239→0.8890
	6%	2.7491	1.6758	(0,4)→(0,3)	0.0350→0.0116	0.7793→0.8425

-The 5% d shock corresponds to the experiment considered in Christiano, Eichenbaum and Rebelo (2009).
- The κ's for the loglinearized solutions are implied values.
-The symbol "→" indicates a change in the value of the variable moving from an equilibrium without a g shock and to one with g shock.

Table 5: Comparison across solution methods: Rotemberg specification, 10% g shock

	d shock (APR)	g multiplier on impact output y	production y^g	ZIRB (S,T)	Resource costs κ on impact	Real unit costs mc on impact
loglinear	2%	0.1269	0.0762	(-,-)→(-,-)	0.0046→0.0009	0.9777→0.9634
	3%	0.8144	0.7586	(0,2)→(-,-)	0.0137→0.0041	0.9359→0.9526
	4%	2.0386	1.9669	(0,4)→(0,1)	0.0392→0.0104	0.8581→0.9264
	5%	3.0858	3.0055	(0,4)→(0,2)	0.0913→0.0259	0.7668→0.8741
	6%	3.3031	3.2178	(0,5)→(0,3)	0.1808→0.0567	0.6719→0.8050
nonlinear	2%	0.0899	-0.0620	(-,-)→(-,-)	0.0037→0.0007	0.9826→0.9631
	3%	0.3737	0.1159	(0,2)→(-,-)	0.0084→0.0032	0.9668→0.9574
	4%	1.2079	0.6416	(0,3)→(0,0)	0.0188→0.0073	0.9300→0.9484
	5%	1.8319	0.8304	(0,4)→(0,2)	0.0350→0.0144	0.8934→0.9249
	6%	2.2321	0.8008	(0,5)→(0,3)	0.0553→0.0257	0.8620→0.8966

-The 5% d shock corresponds to the experiment considered in Christiano, Eichenbaum and Rebelo (2009).
- The κ's for the loglinearized solutions are implied values.
-The symbol "→" indicates a change in the value of the variable moving from an equilibrium without a g shock and to one with g shock.

Table 6: Shorter duration (5 periods) for 1% g shock

Specification	solution method	g multiplier on impact output y	production y^g	ZIRB (S,T)	Resource costs κ on impact	Real unit costs mc on impact
Calvo	loglinear	2.9115	2.8488	(0,4)→(0,4)	0.1558→0.1434	0.7669→0.7769
	nonlinear	2.4116	1.7052	(0,4)→(0,4)	0.0192→0.0177	0.8239→0.8302
Rotemberg	loglinear	2.9128	2.8501	(0,4)→(0,4)	0.0913→0.0843	0.7668→0.7768
	nonlinear	2.1673	1.0609	(0,4)→(0,4)	0.0350→0.0327	0.8934→0.8978

-d shock is set to 5%.

Table 7: Shorter duration (5 periods) for 10% g shock

Specification	solution method	g multiplier on impact output y	production y^g	ZIRB (S,T)	Resource costs κ on impact	Real unit costs mc on impact
Calvo	loglinear	2.8788	2.8031	(0,4)→(0,4)	0.1558→0.0577	0.7669→0.8681
	nonlinear	2.5636	1.9649	(0,4)→(0,4)	0.0192→0.0063	0.8239→0.8982
Rotemberg	loglinear	2.8801	2.8044	(0,4)→(0,4)	0.0913→0.0355	0.7668→0.8681
	nonlinear	2.2178	1.2112	(0,4)→(0,3)	0.0350→0.0142	0.8934→0.9440

-d shock is set to 5%.

Figure 1: Eggertson-Woodford model in steady state

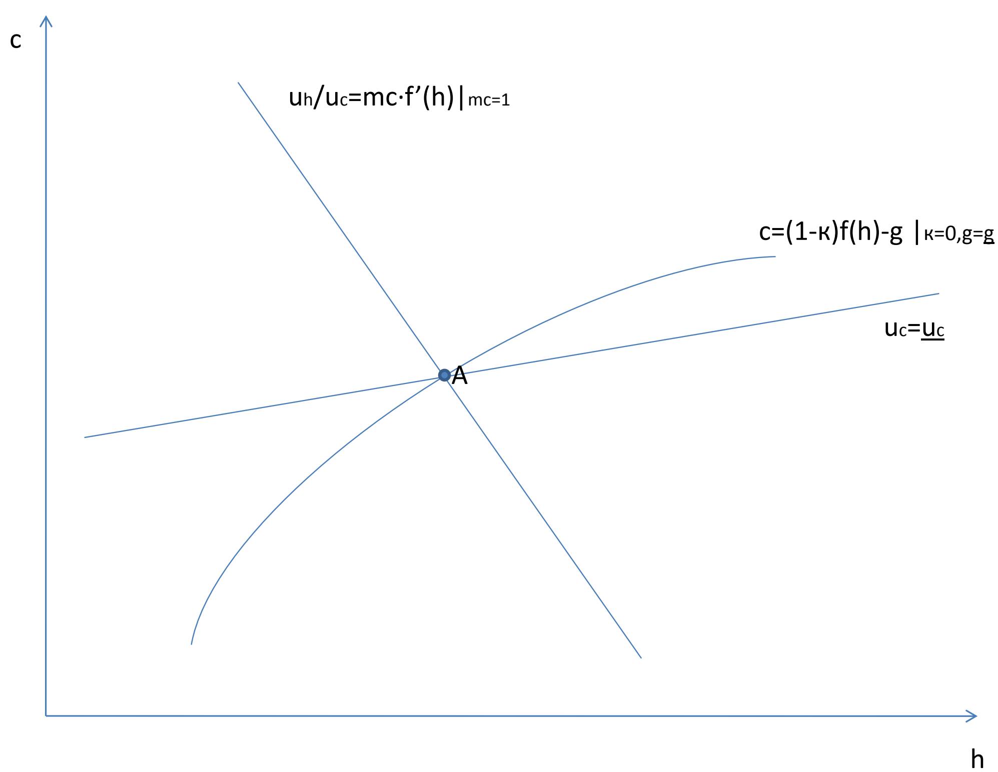


Figure 2: Eggertson-Woodford model with fixed κ and g

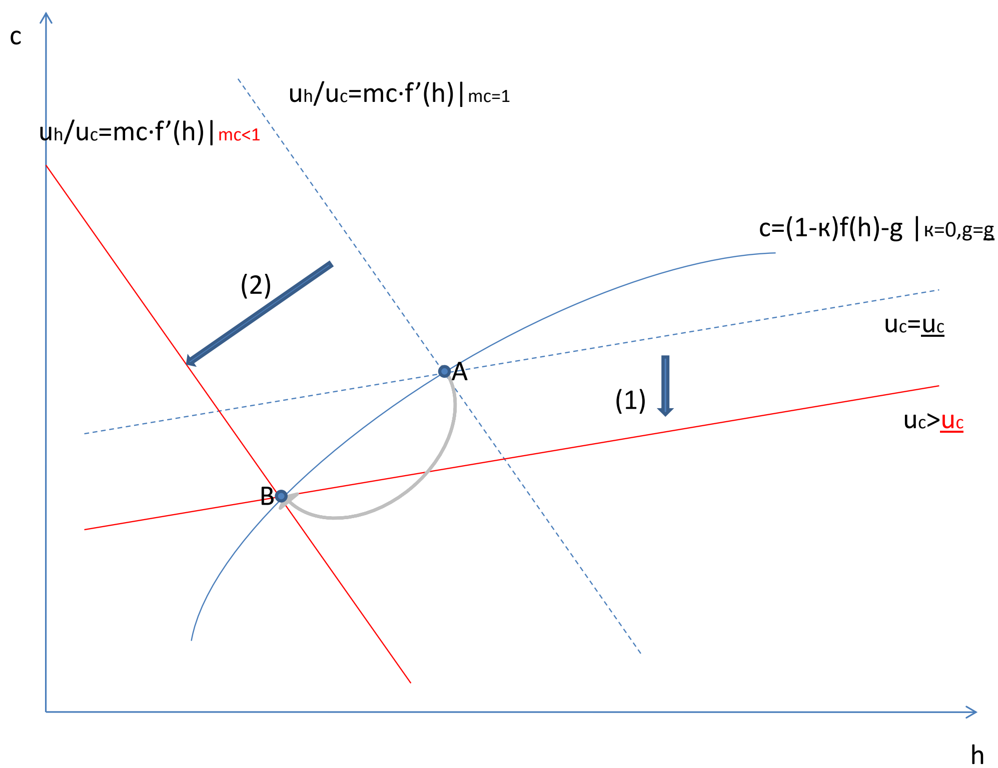


Figure 3: Eggertson-Woodford model with variable κ and g

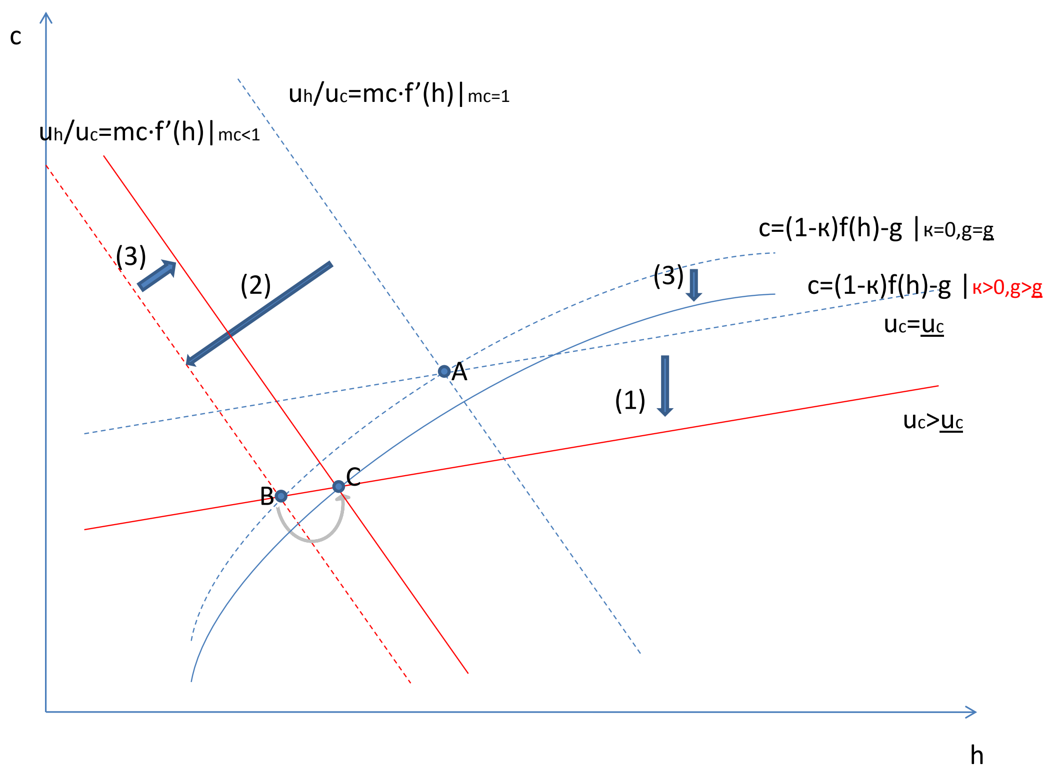
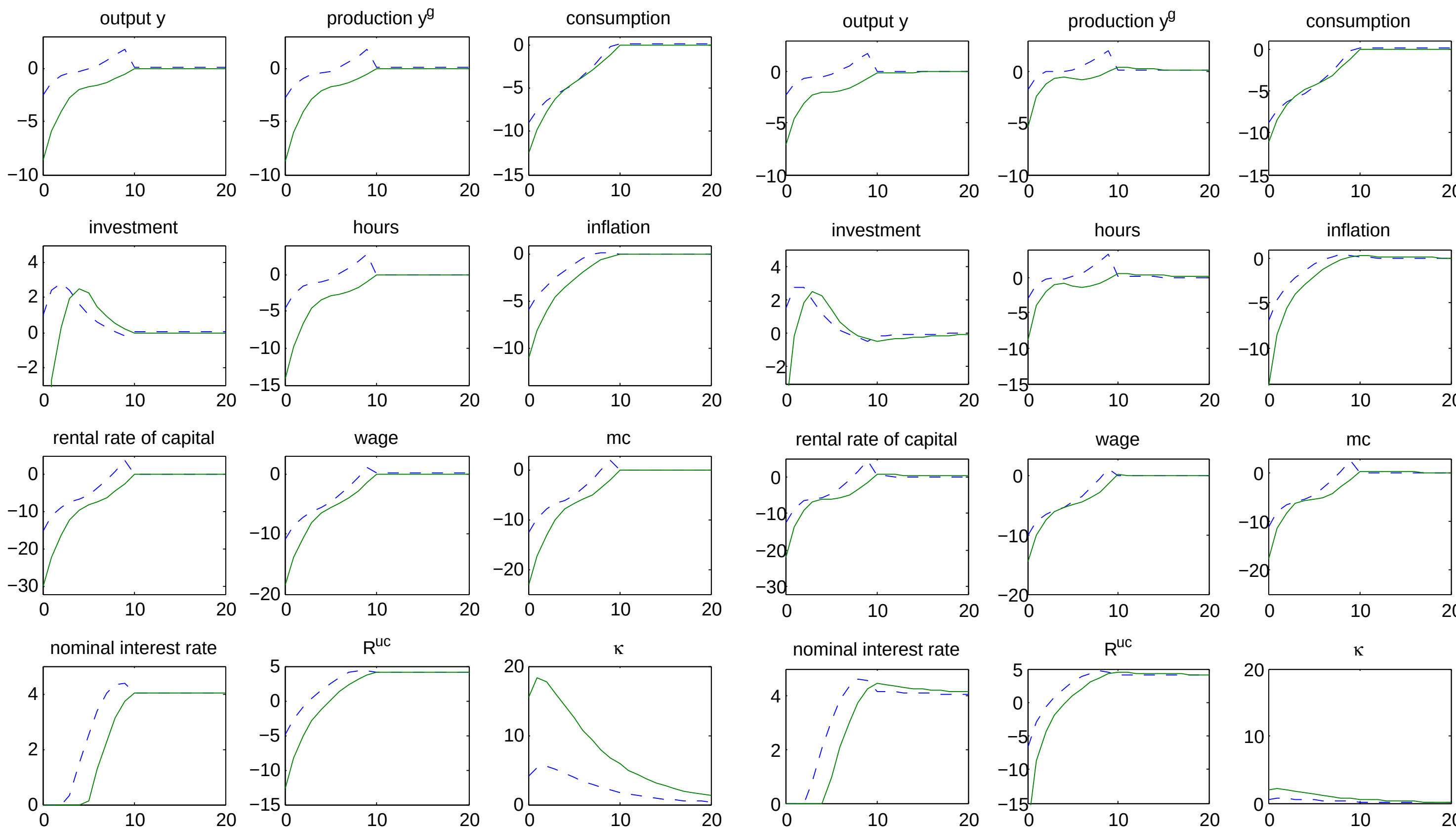


Figure 4: Calvo specification, 10% g shock, 5% discount factor shock

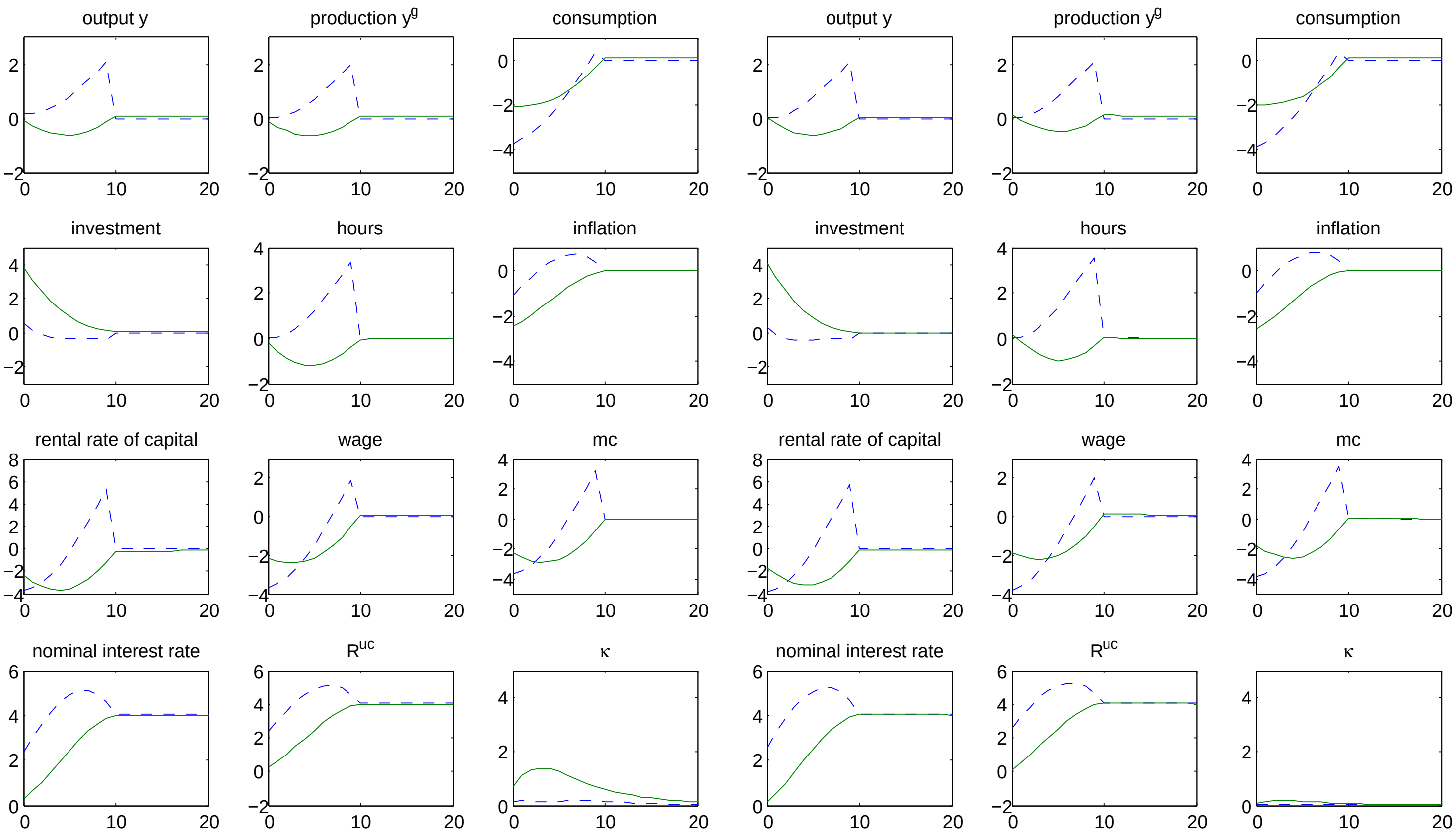


- Loglinear economy on the left panel; Non-linear economy on the right.

- Variables are expressed in terms of percentage deviations from the steady state, except for inflation rate, nominal interest rate, and $R^{uc} = (1 + R_{ss})\left(\frac{1+\pi}{1+\pi_{ss}}\right)^{\rho_\pi} - 1$ which are expressed in terms of levels in annualized percentages.

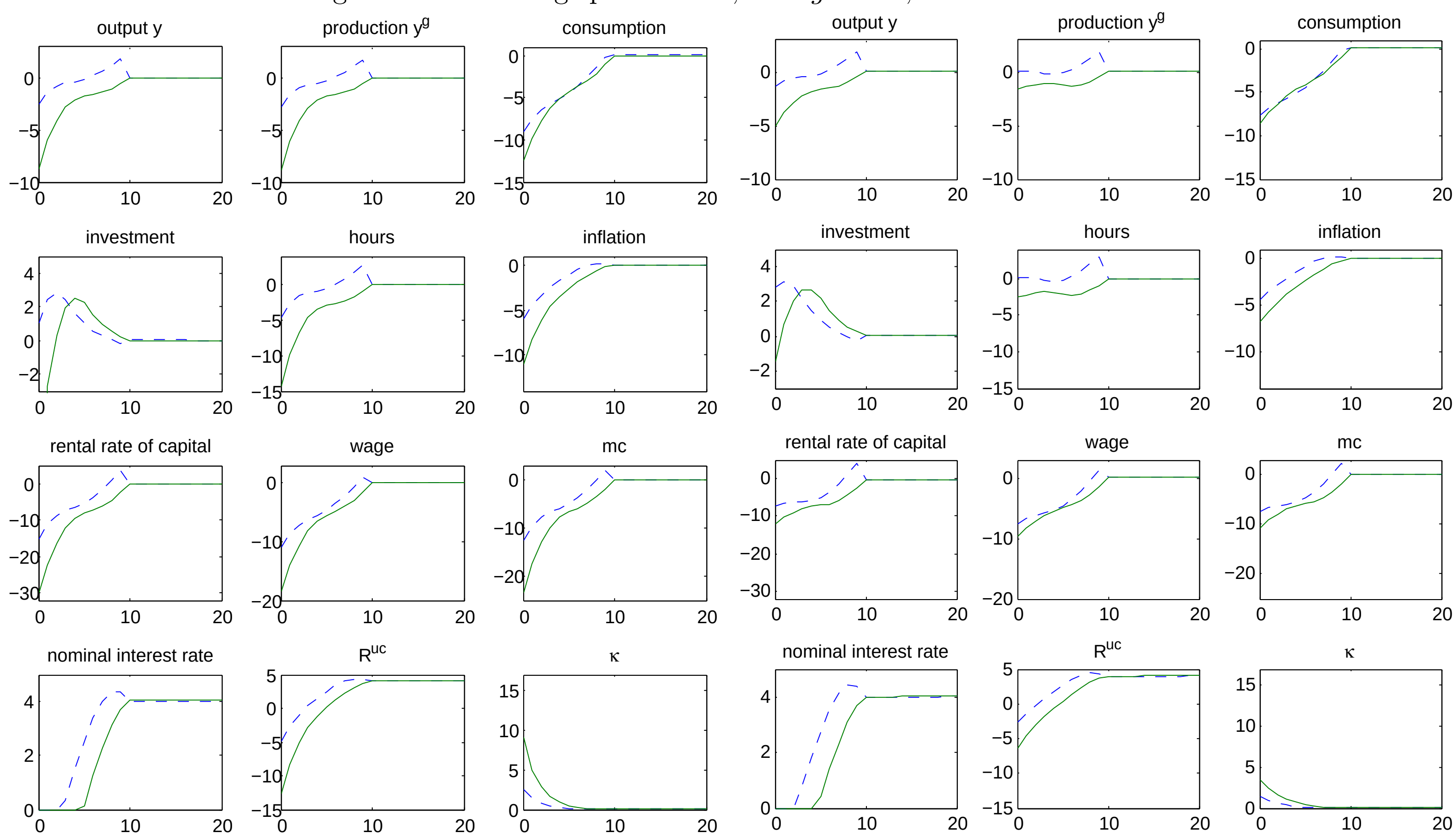
- Solid lines are responses to the preference discount factor, and dashed lines show how the responses change when g is perturbed at the same time.

Figure 5: Calvo specification, 10% g shock, 2% discount factor shock



- Loglinear economy on the left panel; Non-linear economy on the right.
- Variables are expressed in terms of percentage deviations from the steady state, except for inflation rate, nominal interest rate, and $R^{uc} = (1 + R_{ss})\left(\frac{1+\pi}{1+\pi_{ss}}\right)^{\rho_\pi} - 1$ which are expressed in terms of levels in annualized percentages.
- Solid lines are responses to the preference discount factor, and dashed lines show how the responses change when g is perturbed at the same time.

Figure 6: Rotemberg specification, 10% g shock, 5% discount factor shock



- Loglinear economy on the left panel; Non-linear economy on the right.

- Variables are expressed in terms of percentage deviations from the steady state, except for inflation rate, nominal interest rate, and $R^{uc} = (1 + R_{ss})\left(\frac{1+\pi}{1+\pi_{ss}}\right)^{\rho_\pi} - 1$ which are expressed in terms of levels in annualized percentages.

- Solid lines are responses to the preference discount factor, and dashed lines show how the responses change when g is perturbed at the same time.